

VISIT...

LANZAROTE  
*Caliente*.COM

# 7

## Design of Synchronous Generators

---

|      |                                                                                                                                                                                                     |      |
|------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------|
| 7.1  | Introduction .....                                                                                                                                                                                  | 7-2  |
| 7.2  | Specifying Synchronous Generators for Power Systems .....                                                                                                                                           | 7-2  |
|      | The Short-Circuit Ratio (SCR) • SCR and $x_d'$ Impact on Transient Stability • Reactive Power Capability and Rated Power Factor • Excitation Systems and Their Ceiling Voltage                      |      |
| 7.3  | Output Power Coefficient and Basic Stator Geometry .....                                                                                                                                            | 7-10 |
| 7.4  | Number of Stator Slots .....                                                                                                                                                                        | 7-13 |
| 7.5  | Design of Stator Winding .....                                                                                                                                                                      | 7-16 |
| 7.6  | Design of Stator Core .....                                                                                                                                                                         | 7-22 |
|      | Stator Stack Geometry                                                                                                                                                                               |      |
| 7.7  | Salient-Pole Rotor Design .....                                                                                                                                                                     | 7-28 |
| 7.8  | Damper Cage Design .....                                                                                                                                                                            | 7-31 |
| 7.9  | Design of Cylindrical Rotors .....                                                                                                                                                                  | 7-32 |
| 7.10 | The Open-Circuit Saturation Curve .....                                                                                                                                                             | 7-37 |
| 7.11 | The On-Load Excitation mmf $F_{1R}$ .....                                                                                                                                                           | 7-42 |
|      | Potier Diagram Method • Partial Magnetization Curve Method                                                                                                                                          |      |
| 7.12 | Inductances and Resistances .....                                                                                                                                                                   | 7-47 |
|      | The Magnetization Inductances $L_{ad}$ $L_{aq}$ • Stator Leakage Inductance $L_{sl}$                                                                                                                |      |
| 7.13 | Excitation Winding Inductances .....                                                                                                                                                                | 7-50 |
| 7.14 | Damper Winding Parameters .....                                                                                                                                                                     | 7-52 |
| 7.15 | Solid Rotor Parameters .....                                                                                                                                                                        | 7-54 |
| 7.16 | SG Transient Parameters and Time Constants .....                                                                                                                                                    | 7-55 |
|      | Homopolar Reactance and Resistance                                                                                                                                                                  |      |
| 7.17 | Electromagnetic Field Time Harmonics .....                                                                                                                                                          | 7-59 |
| 7.18 | Slot Ripple Time Harmonics .....                                                                                                                                                                    | 7-61 |
| 7.19 | Losses and Efficiency .....                                                                                                                                                                         | 7-63 |
|      | No-Load Core Losses of Excited SGs • No-Load Losses in the Stator Core End Stacks • Short-Circuit Losses • Third Flux Harmonic Stator Teeth Losses • No-Load and On-Load Solid Rotor Surface Losses |      |
| 7.20 | Exciter Design Issues .....                                                                                                                                                                         | 7-75 |
|      | Excitation Rating • Sizing the Exciter • Note on Thermal and Mechanical Design                                                                                                                      |      |
| 7.21 | Optimization Design Issues .....                                                                                                                                                                    | 7-78 |
| 7.22 | Generator/Motor Issues .....                                                                                                                                                                        | 7-80 |
| 7.23 | Summary .....                                                                                                                                                                                       | 7-80 |
|      | References .....                                                                                                                                                                                    | 7-84 |

## 7.1 Introduction

Most synchronous generator power is transmitted through power systems to various loads, but there are various stand-alone applications, too.

In this chapter, the design of synchronous generators (SGs) connected to a power system is dealt with in some detail.

The successful design and operation of an SG depends heavily on agreement between the SG manufacturer and user in regard to technical requirements (specifications). Published standards such as American National Standards Institute (ANSI) C50.13 and International Electrotechnical Commission (IEC) 34-1 contain these requirements for a broad class of SGs. The Institute of Electrical and Electronics Engineers (IEEE) recently launched two new, consolidated standards for high-power SGs [1]:

- C50.12 for large salient pole generators
- C50.13 for cylindrical rotor large generators

The liberalization of electricity markets led, in the past 10 years, to the gradual separation of production, transport, and supply of electrical energy. Consequently, to provide for safe, secure, and reasonable cost supply, formal interface rules — grid codes — were put forward recently by private utilities around the world.

Grid codes do not align in many cases with established standards, such as IEEE and ANSI. Some grid codes exceed the national and international standards “Requirements on Synchronous Generators.” Such requirements may impact unnecessarily on generator costs, as they may not produce notable benefits for power system stability [2].

Harmonization of international standards with grid codes becomes necessary, and it is pursued by the joint efforts of SG manufacturers and interconnectors [3] to specify the turbogenerator and hydrogenerator parameters. Generator specifications parameters are, in turn, related to the design principles and, ultimately, to the costs of the generator and of its operation (losses, etc.).

In this chapter, a discussion of turbogenerator specifications as guided by standards and grid codes is presented in relation to fundamental design principles. Hydrogenerators pose similar problems in power systems, but their power share is notably smaller than that of turbogenerators, except for a few countries, such as Norway. Then, the design principles and a methodology for salient pole SGs and for cylindrical rotor generators, respectively, with numerical examples, are presented in considerable detail.

Special design issues related to generator motors for pump-storage plants or self-starting turbogenerators are treated in a dedicated paragraph.

## 7.2 Specifying Synchronous Generators for Power Systems

The turbogenerators are at the core of electric power systems. Their prime function is to produce the active power. However, they are also required to provide (or absorb) reactive power both, in a refined controlled manner, to maintain frequency and voltage stability in the power system (see [Chapter 6](#)). As the control of SGs becomes faster and more robust, with advanced nonlinear digital control methods, the parameter specification is about to change markedly.

### 7.2.1 The Short-Circuit Ratio (SCR)

The short-circuit ratio (SCR) of a generator is the inverse ratio of saturated direct axis reactance in per unit (P.U.):

$$SCR = \frac{1}{x_{d(sat)}} \quad (7.1)$$

The SCR has a direct impact on the static stability and on the leading (absorbed) reactive power capability of the SG. A larger SCR means a smaller  $x_{d(sat)}$  and, almost inevitably, a larger airgap. In turn,

this requires more ampere-turns (magnetomotive force [mmf]) in the field winding to produce the same apparent power.

As the permissible temperature rise is limited by the SG insulation class (class B, in general,  $\Delta T = 130^\circ$ ), more excitation mmf means a larger rotor volume and, thus, a larger SG.

Also, the SCR has an impact on SG efficiency. An increase of SCR from 0.4 to 0.5 tends to produce a 0.02 to 0.04% reduction in efficiency, while it increases the machine volume by 5 to 10% [3].

The impact of SCR on SG static stability may be illustrated by the expression of electromagnetic torque  $t_e$  P.U. in a lossless SG connected to a infinite power bus:

$$t_e \approx SCR \cdot E_0 \cdot V_1 \sin \delta \quad (7.2)$$

The larger the SCR, the larger the torque for given no-load voltage ( $E_0$ ), terminal voltage  $V_1$ , and power angle  $\delta$  (between  $E_0$  and  $\Delta V_1$  per phase). If the terminal voltage decreases, a larger SCR would lead to a smaller power angle  $\delta$  increase for given torque (active power) and given field current.

If the transmission line reactance — including the generator step-up transformer — is  $x_e$ , and  $V_1$  is now replaced by the infinite grid voltage  $V_g$  behind  $x_e$ , the generator torque  $t'_e$  is as follows:

$$t'_e = SCR \times E_0 \times V_g \times \frac{\sin \delta'}{(1 + x_e / x_d)} \quad (7.3)$$

The power angle  $\delta'$  is the angle between  $E_0$  of the generator and  $V_g$  of the infinite power grid. The impact of improvement of a larger SCR on maximum output is diminished as  $x_e/x_d$  increases.

Increasing SCR from 0.4 to 0.5 produces the same maximum output if the transmission line reactance ratio  $x_e/x_d$  increases from 0.17 to 0.345 at a leading power factor of 0.95 and 85% rated megawatt (MW) output.

Historically, the trend has been toward lower SCRs, from 0.8 to 1.0, 70 years ago, to 0.58 to 0.65 in the 1960s, and to 0.5 to 0.4 today. Modern — fast response — excitation systems compensate for the apparent loss of static stability grounds. The lower SCRs mean lower generator volumes, losses, and costs.

## 7.2.2 SCR and $x_d'$ Impact on Transient Stability

The critical clearing time of a three-phase fault on the high-voltage side of the SG step-up transformer is a representative performance index for the transient stability limits of the SG tied to an infinite bus bar.

The transient  $d$ -axis reactance  $x_d'$  (in P.U.) takes the place of  $x_d$  in Equation 7.3 to approximate the generator torque transients before the fault clearing. In the case in point,  $x_e = x_{Tsc}$  is the short-circuit reactance (in P.U.) of the step-up transformer. A lower  $x_d'$  allows for a larger critical clearing time and so does a large inertia. Air-cooled SGs tend to have a larger inertia/MW than hydrogen-cooled SGs, as their rotor size is relatively larger and so is their inertia.

## 7.2.3 Reactive Power Capability and Rated Power Factor

A typical family of V curves is shown in Figure 7.1. The reactive power capability curve (Figure 7.2) and the V curves are more or less equivalent in reflecting the SG capability to deliver active and reactive power, or to absorb reactive power until the various temperature limitations are met (Chapter 5). The rated power factor determines the delivered/lagging reactive power continuous rating at rated active power of the SG.

The lower the rated (lagging) power factor, the larger the MVA per rated MW. Consequently, the excitation power is increased, and the step-up transformer has to be rated higher. The rated power factor is generally placed in the interval 0.9 to 0.95 (overexcited) as a compromise between generator initial and loss capitalized costs and power system requirements. Lower values down to 0.85 (0.8) may be found

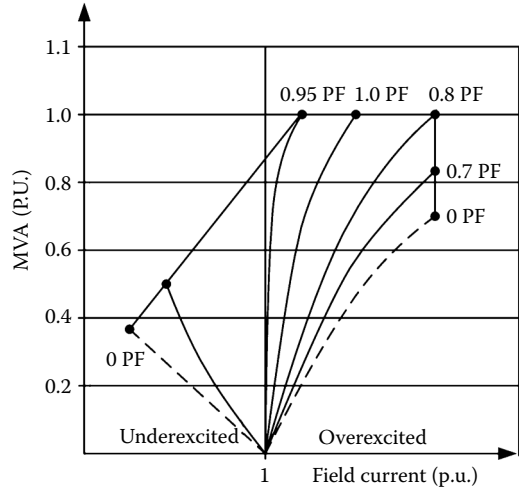


FIGURE 7.1 Typical V curve family.

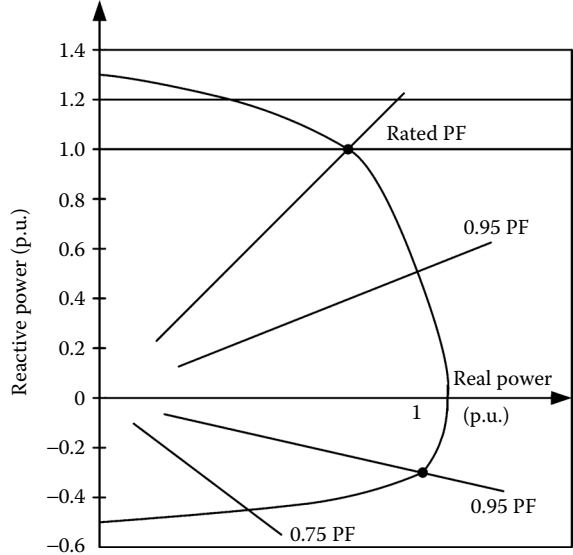


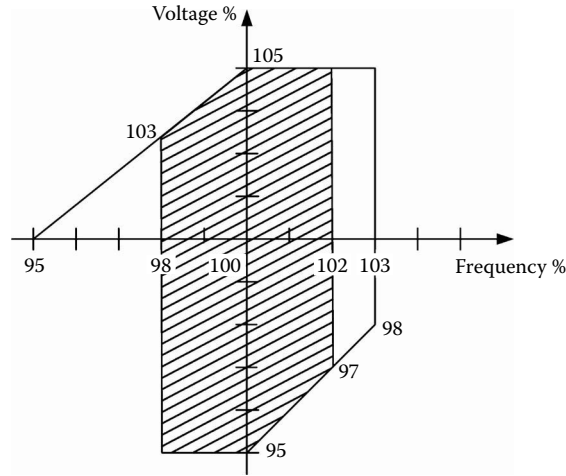
FIGURE 7.2 Reactive power capability curve.

in air (hydrogen)-cooled SGs. The minimum underexcited rated power factor is 0.95 at rated active power. The maximum absorbed (leading) reactive power limit is determined by the SCR and corresponds to maximum power angle and to end stator core overtemperature limit.

### 7.2.4 Excitation Systems and Their Ceiling Voltage

Fast control of excitation current is needed to preserve SG transient stability and control its voltage. Higher ceiling excitation voltage, corroborated with low electrical time constants in the excitation system, provides for fast excitation current control.

Today's ceiling voltages are in the range of 1.6 to 3.0 P.U. There is a limit here dictated by the effect of magnetic saturation, which makes ceiling voltages above 1.6 to 2.0 P.U. hardly practical. This is more so as higher ceiling voltage means sizing the insulation system of the exciter or the rating of the static exciter voltage for maximum ceiling voltage at notably larger exciter costs.



**FIGURE 7.3** Voltage/frequency operation.

The debate over which is best — the alternating current (AC) brushless exciter or static exciter (which is specified also with a negative ceiling voltage of  $-1.2$  to  $1.5$  P.U.) is still not over. A response time of 50 msec in “producing” the maximum ceiling voltage is today fulfilled by the AC brushless exciters, but faster response times are feasible with static exciters. However, during system faults, the AC brushless exciter is not notably disturbed, as it draws its input from the kinetic energy of the turbine-generator unit.

In contrast, the static exciter is fed from the exciter transformer which is connected, in general, at SG terminals, and seldom to a fully independent power source. Consequently, during faults, when the generator terminal voltage decreases, to secure fast, undisturbed excitation current response, a higher voltage ceiling ratio is required. Also, existing static exciters transmit all power through the brush–slip–ring mechanical system, with all the limitations and maintenance incumbent problems.

#### 7.2.4.1 Voltage and Frequency Variation Control

As detailed in Chapter 6, the SG has to deliver active and reactive power with designed speed and voltage variations. The size of the generator is related to the active power (frequency) and reactive power (voltage) requirements. Typical such practical requirements are shown in Figure 7.3.

In general, SGs should be thermally capable of continuous operation within the limits of the  $P/Q$  curve (Figure 7.2) over the ranges of  $\pm 5\%$  in voltage, but not necessarily at the power level typical for rated frequency and voltage. Voltage increase, accompanied by frequency decrease, means a higher increase in the  $V/\omega$  ratio.

The total flux in the machine increases. A maximum of flux increase is considered practical and should be there by design. The SG has to be sized to have a reasonable magnetic saturation level (coefficient) such that the field mmf (and losses) and the core loss are not increased so much as to compromise the thermal constraints in the presence of corresponding adjustments of active and reactive power delivery under these conditions.

To avoid oversizing the SG, the continuous operation is guaranteed only in the hatched area, at most, 47.5 to 52 Hz. In general, the 5% overvoltage is allowed only above rated frequency, to limit the flux increase in the machine to a maximum of 5%. The rather large  $\pm 5\%$  voltage variation is met by SGs with the use of tap changers on the generator step-up transformer (according to IEC standards).

#### 7.2.4.2 Negative Phase Sequence Voltage and Currents

Grid codes tend to restrict the negative sequence voltage component at 1% ( $V_2/V_1$  in percent). Peaks up to 2% might be accepted for short duration by prior agreement between manufacturer and interconnector.

The SGs should be able to withstand such voltage imbalance, which translates into negative sequence currents in the stator and rotor with negative sequence reactance  $x_2 = 0.10$  (the minimum accepted by

the IEC) and a step-up transformer with a reactance  $x_T = 0.15$  P.U. Then, the 1% voltage unbalance translates into a negative sequence current  $i_2$  (P.U. in percent) of

$$i_2 = \frac{v_2}{x_2 + x_T} = \frac{0.01}{0.1 + 0.15} = 0.04 \text{ P.U.} = 4\% \quad (7.4)$$

The SG has to be designed to withstand the additional losses in the rotor damper cage, in the excitation winding, and in the stator winding, produced by the negative sequence stator current. Turbogenerators above 700 MV seem to need explicit amortisseur windings for the scope.

#### 7.2.4.3 Harmonic Distribution

Grid codes specify the voltage total harmonic distortion (THD) at 1.5% and 2% in, respectively, near 400 kV and in the near 275 kV power systems. Proposals are made to raise these values to 3 (3.5)% in the voltage THD. The voltage THD may be converted into current THD and then into an equivalent current for each harmonic, considering that the inverse reactance  $x_2$  may be applied for time harmonics as well.

For the fifth time harmonic, for example, a 3% voltage THD corresponds to a current  $i_5$ :

$$i_5 = \frac{v_5}{5 \cdot (x_2 + x_T)} = \frac{0.03}{5 \cdot (0.1 + 0.15)} = 0.024 \text{ P.U.} \quad (7.5)$$

#### 7.2.4.4 Temperature Basis for Rating

Observable and hot-spot temperature limits appear in IEEE/ANSI standards, but only the former appears in IEC-60034 standards.

In principle, the observable temperature limits have to be set such that the hot-spot temperatures should not go above 130° for insulation class B and 155° for insulation class F.

In practice, one design could meet observable temperatures (in a few spots in the SG) but exceed the hot-spot limits of the insulation class. Or, we may overrestrict the observable temperature, while the hot spot may be well below the insulation class limit.

Also, the rated cold coolant temperature has to be specified if the hot-spot temperature is maintained constant when the cold coolant temperature varies, as for ambient temperature, following SGs where the observable temperature also varies.

Holding one of the two temperature limits as constant, with the cold coolant (ambient) temperature variable, leads to different SG overrating and underrating (Figure 7.4).

It seems reasonable that we need to fix the observable temperature limit for a single cold coolant temperature and calculate the SG MVA capability for different cold coolant (ambient) and hot-spot temperatures. This way, the SG is exploited optimally, especially for the “ambient-following” operation mode.

#### 7.2.4.5 Ambient-Following Machines

SGs that operate for ambient temperatures between −20° and 50° should have permissible generator output power, variable with cold coolant temperatures. Eventually, peak (short-term) and base MVA capabilities should be set at rated power factor (Figure 7.5).

#### 7.2.4.6 Reactances and Unusual Requirements

The already mentioned  $d$ -axis synchronous reactance  $x_d$  and  $d$ -axis transient reactance  $x'_d$  are key factors in defining static and transient stability and maximum leading reactive power rating of SGs. In general practice,  $x_d$  and  $x'_d$  values are subject to agreement between vendors and purchasers of SGs, based on operating conditions (weak or strong power system area exciter performance, etc.).

To limit the peak short-circuit current and circuit breaker rating, it may be considered as appropriate to specify (or agree upon) a minimum value of the subtransient reactances at the saturation level of rated

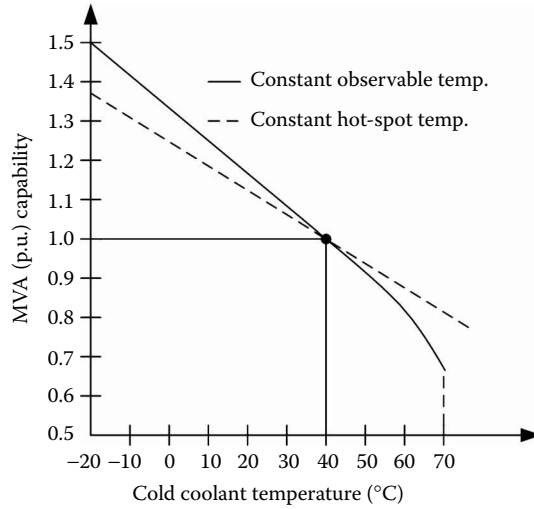


FIGURE 7.4 Synchronous generator millivoltampere rating vs. cold coolant temperature.

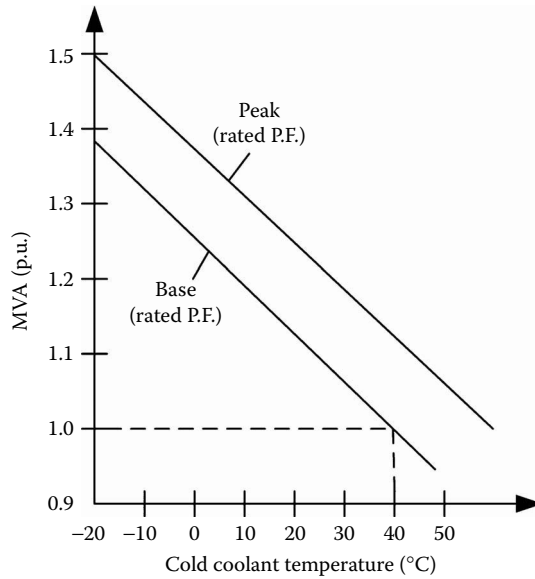


FIGURE 7.5 Ambient following synchronous generator ratings.

voltage. Also, the maximum value of the unsaturated (at rated current) value of transient  $d$  axis reactance  $x_d'$  may be limited based on unsaturated and saturated subtransient and transient reactances, see IEEE 100 [11].

There should be tolerances for these agreed-upon values of  $x_d''$  and  $x_d'$ , positive for the first ( $20 \div 30\%$ ) and negative ( $-20 \div 30\%$ ) for the second.

#### 7.2.4.7 Start-Stop Cycles

The total number of starts is important to specify, as the SG should, by design, prevent cyclic fatigue degradation. According to IEC and IEEE and ANSI trends, it seems that the number of starts should be as follows:



- 3000 for base-load SGs
- 10,000 for peak-load SGs or other frequently cycled units [1]

#### 7.2.4.8 Starting and Operation as a Motor

Combustion turbines generator units may be started with the SG as a motor fed from a static power converter of lower rating, in general. Power electronics rating, drive-train losses, inertia, speed vs. time, and restart intervals have to be considered to ensure that the generator temperatures are all within limits.

Pump-storage hydrogenerator units also have to be started as motors on no-load, with power electronics, or back-to-back from a dedicated generator which accelerates simultaneously with the asynchronous motor starting. The pumping action will force the SG to work as a synchronous motor and the hydraulic turbine-pump and generator-motor characteristics have to be optimally matched to best exploit the power unit in both operation modes.

#### 7.2.4.9 Faulty Synchronization

SGs are also designed to survive without repairs after synchronization with  $\pm 10^\circ$  initial power angle. Faulty synchronization (outside  $\pm 10^\circ$ ) may cause short-duration current and torque peaks larger than those occurring during sudden short-circuits. As a result, internal damage of the SG may result; therefore, inspection for damage is required. Faulty synchronization at  $120^\circ$  or  $180^\circ$  out of phase with a low system reactance (infinite) bus might require partial rewind of the stator and extensive rotor repairs. Special attention should be paid to these aspects from design stage on.

#### 7.2.4.10 Forces

Forces in an SG occur due to the following:

- System faults
- Thermal expansion cycles
- Double-frequency (electromagnetic) running forces

The relative number of cycles for peaking units (one start per day for 30 yr) is shown in Figure 7.6 [4], together with the force level.

For system faults (short-circuit, faulty, or successful synchronization), forces have the highest level (100:1). The thermal expansion forces have an average level (1:1), while the double-frequency running forces are the smallest in intensity (1:10). A base load unit would encounter a much smaller thermal expansion cycle count.

The mechanical design of an SG should manage all these forces and secure safe operation over the entire anticipated operation life of the SG.

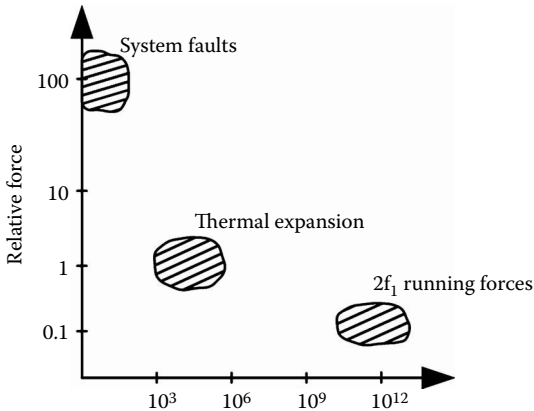


FIGURE 7.6 Forces cycles.

#### 7.2.4.11 Armature Voltage

In principle, the armature voltage may vary in a 2-to-1 ratio without having to change the magnetic flux or the armature reaction mmf, that is, for the same machine geometry.

Choosing the voltage should be the privilege of the manufacturer, to enable him enough freedom to produce the best designs for given constraints. The voltage level determines the insulation between the armature winding and the slot walls in an indirectly cooled SG. This is not so in direct-cooled stator (rotor) windings, where the heat is removed through a cooling channel located in the slots. Consequently, a direct-cooled SG may be designed for higher voltages (say 28 kV instead of 22 kV) without paying a high price in cooling expenses.

However, for air-cooled generators, higher voltage may influence the Corona effect. This is not so in hydrogen-cooled SGs because of the higher Corona start voltage.

#### 7.2.4.12 Runaway Speed

The runaway speed is defined as the speed the prime mover may be allowed to have if it is suddenly unloaded from full (rated) load. Steam (or gas) turbines are, in general, provided with quick-action speed governors set to trip the generator at 1.1 times the rated speed. So, the runaway speed for turbogenerators may be set at 1.25 P.U. speed. For water (hydro) turbines, the runaway speeds are much higher (at full gate opening):

- 1.8 P.U. for Pelton (impulse) turbines (SGs)
- 2.0 to 2.2 P.U. for Francis turbines (SGs)
- 2.5 to 2.8 P.U. for Kaplan (reaction) turbines (SGs)

The SGs are designed to withstand mechanical stress at runaway speeds. The maximum peripheral speed is about 140 to 150 m/sec for salient-pole SGs and 175 to 180 m/sec for turbogenerators. The rotor diameter design is limited by this maximum peripheral speed.

The turbogenerators are built today in only two-pole configurations, either at 50 Hz or at 60 Hz.

#### 7.2.4.13 Design Issues

SG design deals with many issues. Among the most important issues are the following:

- Output coefficient and basic stator geometry
- Number of stator slots
- Design of stator winding
- Design of stator core
- Salient-pole rotor design
- Cylindrical rotor design
- Open-circuit saturation curve
- Field current at full load
- Stator leakage inductance, resistance, and synchronous reactance calculation
- Losses and efficiency calculation
- Calculation of time constant and transient and subtransient reactance
- Cooling system and thermal design
- Design of brushes and slip-rings (if any)
- Design of bearings
- Brakes and jacks design
- Exciter design

Currently, design methodologies of SGs are put in computer codes, and they may contain optimization stages and interface with finite element software for the refined calculation of electromagnetic thermal and mechanical stress, either for verification or for the final geometrical optimization design stage.

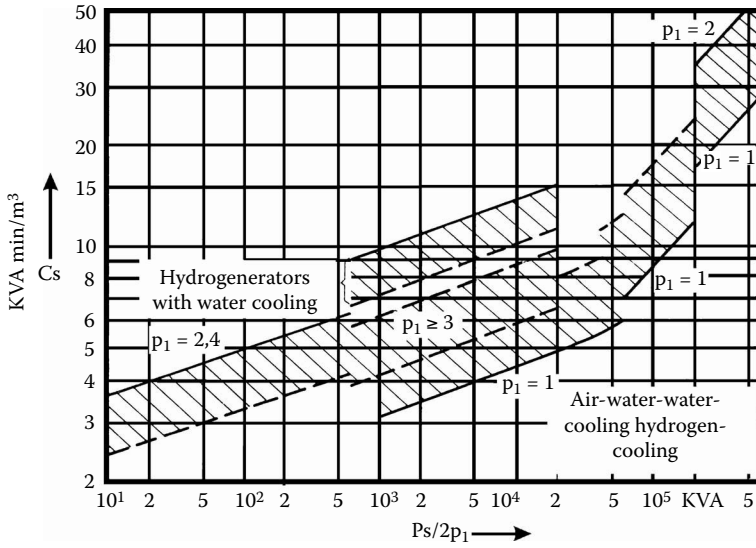


FIGURE 7.7 Output power coefficient for synchronous generators.

### 7.3 Output Power Coefficient and Basic Stator Geometry

The output coefficient  $C$  is defined as the SG kilovoltampere per cubic meter of rotor volume. The value of  $C$  (kilovoltampere per cubic meter) depends on machine power/pole, the number of pole pairs  $p_1$ , and the type of cooling, and it is often based on past experience (Figure 7.7).

The output power coefficient  $C$  may be expressed in terms of machine magnetic and electric loadings, starting from the electromagnetic power  $P_{elm}$ :

$$P_{elm} = 3 \cdot \frac{\omega_1}{\sqrt{2}} \cdot (W_1 \cdot K_{W1}) \cdot \Phi_1 \cdot I_1 \quad ; \quad \omega_1 = 2\pi p_1 \cdot n_n \quad (7.6)$$

The ampere-turns per meter, or the electric specific loading ( $A_1$ ), is as follows:

$$A_1 = \frac{6 \cdot W_1 \cdot I_1}{\pi \cdot D \cdot l_i} \quad (\text{A/m}) \quad ; \quad K_{W1} - \text{winding factor} \quad (7.7)$$

with  $l_i$  the ideal stator stack length and  $D$  the rotor (or stator bore) diameter.

The flux per pole  $\Phi_1$  is

$$\Phi_1 = \frac{2}{\pi} \cdot B_{g1} \cdot \frac{\pi \cdot D}{2p_1} \cdot l_i \quad (7.8)$$

Making use of Equation 7.7 and Equation 7.8 in Equation 7.6 yields

$$P_{elm} = \frac{\pi^2}{\sqrt{2}} \cdot K_{W1} \cdot A_1 \cdot B_{g1} \cdot l_i \cdot n_1 \cdot D^2 = C \cdot D^2 \cdot l_i \cdot n_n \quad (7.9)$$

So,

$$C = \frac{\pi^2}{\sqrt{2}} \cdot K_{w1} \cdot A_1 \cdot B_{g1} \quad (7.10)$$

The airgap flux density  $B_{g1}$  relates to magnetic specific loading (saturation level), while  $A_1$  defines the electric specific loading.  $C$  is not quite equal to power/rotor volume but is proportional to it. The proportionality coefficient is  $\pi/4$ .

Going further, we might define the average shear stress on rotor  $f_t$  (specific tangential force in Newton per square meter [N/m<sup>2</sup>] or Newton per square centimeter [N/cm<sup>2</sup>]):

$$f_t = \frac{F_t}{\pi \cdot D \cdot l_i} = \frac{T_{elm} \cdot \left(\frac{2}{D}\right)}{\pi \cdot D \cdot l_i} = \frac{P_{elm}}{2\pi n_1 \cdot \pi \cdot \frac{D}{2} \cdot D \cdot l_i} = \frac{1}{\pi^2} C \quad (7.11)$$

So, the power output coefficient  $C$  is proportional to specific tangential force  $f_t$  exerted on the rotor exterior surface by the electromagnetic torque, with  $C$  given in voltampere per cubic meter [VA/m<sup>3</sup>],  $f_t$  comes into Newton per square meter [N/m<sup>2</sup>]. In general,  $C$  is given in kilovoltampere per cubic meter [kVA/m<sup>3</sup>].

As seen in Figure 7.7,  $C$  is given as a function of power per pole:  $P_s/2p_1$  [5]. Direct water cooling in turbogenerators ( $2p_1 = 2, 4$ ) allows for the highest output power coefficient.

The provisional rotor diameter  $D$  of SGs is limited by the maximum peripheral speed (140 to 150 m/sec) with 44 to 55 kg/mm<sup>2</sup> yield point, typical rotor core materials. This maximum peripheral speed  $U_{max}$  is to be reached at the runaway speed  $n_{max}$ , set by design as discussed earlier:

$$U_{max} = \pi \cdot D_{max} \cdot n_n \cdot \frac{n_{max}}{n_n} \quad (7.12)$$

For hydrogenerators  $n_{max}/n_n$  is much larger than the value for turbogenerators.

It is imperative that the chosen diameter gives the desired flywheel effect required by the turbine design. As already discussed in Chapter 5, the inertia constant  $H$  in seconds is

$$H = \frac{J(\omega_1/p_1)^2}{2S_n} \quad (7.13)$$

where

$J$  = the rotor inertia (in kilogram  $\times$  square meter)

$S_n$  = the rated apparent power in voltampere

$H$  = defined in relation to the maximum speed increase allowed until the speed governor closes the fuel (water) input

In general,

$$\frac{\Delta n_{max}}{n_n} \approx \sqrt{\frac{T_{GV}}{H} + 1} \quad (7.14)$$

with  $T_{GV}$  equal to the speed governor (gate) time constant in seconds.

For hydrogenerators,

$$\frac{\Delta n_{\max}}{n_n} < 0.3 - 0.4 \quad (7.15)$$

$T_{GV}$  for hydrogenerators is in the order of 5 to 8 sec. For turbogenerators,  $T_{GV}$  and  $\Delta n_{\max}/n_n$  are notably smaller (<0.1 to 0.15).  $H$  for hydrogenerators varies in the interval from 3 to 8 sec above 1 MVA per unit.  $H$  is often stated as  $GD_{ig}^2$  ( $\text{kg} \cdot \text{m}^2$ ), where  $D_{ig}$  is twice the gyration radius of the rotor, and  $G$  is the rotor weight in kilogram:

$$H \approx \frac{1.37 \times 10^{-6} GD_{ig}^2 n_n^2}{S_n (\text{kVA})} \quad (\text{kWsec/kVA}) \quad (7.16)$$

Approximately,

$$GD_{ig}^2 \approx \frac{\pi \gamma_{\text{iron}}}{8} \left[ 1 - \left( \frac{D_{ir}}{D} \right)^2 \right] \cdot D^4 \cdot l_p; \quad l_p - \text{rotor length} \quad (7.17)$$

where

- $\gamma_{\text{iron}}$  = the iron specific weight ( $\text{kg/m}^3$ )
- $D_{ir}$  = the interior rotor diameter
- $D_{ir}$  = zero in turbogenerators

$GD_{ig}^2$  may be specified in tonne  $\times$  square meter. Alternatively,  $H$  in seconds may be specified or calculated from Equation 7.14 with  $T_{GV}$  and  $\Delta n_{\max}/n_n$  already specified.

With the rotor diameter provisional upper limit from Equation 7.12, the length of the stator core stack  $l_i$  may be calculated from Equation 7.9 if  $P_{elm}$  is replaced by  $S_n$ . Then, with  $GD_{ig}^2$  or  $H$  given, from Equation 7.16 and Equation 7.17 and with length  $l_p \approx l_i$ , the internal rotor interior diameter  $D_{ir} < D$  may be calculated.

The pole pitch may also be computed:

$$\tau \approx \frac{\pi D}{2 p_1} \quad ; \quad p_1 = \frac{f_n}{n_n} \quad (7.18)$$

The ratio  $l_i/\tau$  has to be placed in a certain interval to secure low enough stator copper losses, as the end-connections length of the stator is proportional to the pole pitch  $\tau$ . Generally,

$$\begin{aligned} \lambda = l_i/\tau &= 1 \div 4 \quad \text{for } p_1 = 1 \\ &= 0.5 \div 2.5 \quad \text{for } p_1 > 1 \end{aligned} \quad (7.19)$$

The intervals for  $\lambda$  are rather large, leaving the designer with ample freedom. Though optimization design may be performed, it is good to have a good design start, so  $\lambda$  has to be in the intervals suggested by Equation 7.19.

With the output power coefficient  $C$  given by Equation 7.10, and based on past experience, the airgap flux density fundamental  $B_{gl}$  is as follows:

- $B_{gl} = 0.75 - 1.05$  T for cylindrical rotor SGs
- $B_{gl} = 0.80 - 1.05$  T for salient pole rotor SGs

Correspondingly, with  $C$  from Figure 7.7, the linear current loading  $A$  ( $\text{A/m}$ ) intervals may be calculated for various cooling methods. The orientative design current densities intervals may also be specified (Table 7.1).

**TABLE 7.1** Orientative Electric “Stress” Parameters

|                                                        | Indirect Air Cooling | Indirect Hydrogen Cooling                                                                 | Direct Cooling    |
|--------------------------------------------------------|----------------------|-------------------------------------------------------------------------------------------|-------------------|
| A (kA/m)                                               | 30–80                | 90–120                                                                                    | 160–200           |
| Stator current density $j_{\cos}$ (A/mm <sup>2</sup> ) | 3–6                  | 4–7                                                                                       | 7–10              |
| Rotor current density $j_{\cos}$ (A/mm <sup>2</sup> )  | 3–5                  | 3–5                                                                                       | For water<br>6–13 |
|                                                        |                      | With stator and rotor direct cooling: (13–18)<br>A/mm <sup>2</sup> and A = (250–300) kA/m |                   |

## 7.4 Number of Stator Slots

The first requirement in determining the number of stator slots is to produce symmetrical (balanced) three-phase electromagnetic fields (emfs).

For  $q$  equal to the integer number of slots per pole and phase, the number of stator slots  $N_s$  is

$$N_s = 2p_1 \cdot q \cdot m \quad ; \quad m = 3 \text{ phases}; \quad p_1 - \text{pole pairs} \quad (7.20)$$

A larger integer  $q$  is typical for turbogenerators ( $2p_1 = 2, 4$ ):  $q > (4 \text{ to } 6)$ . For low-speed generators,  $q$  may be as low as three but not less. For  $q < 3, 4$  and for large power hydrogenerators, a fractionary  $q$  winding is adopted:

$$N_s = 2p_1 \cdot \left( b + \frac{c}{d} \right) \cdot 3 \quad ; \quad q = b + \frac{c}{d} \quad (7.21)$$

To secure balanced emfs, the slot pitch number  $x$  between the start of phases A and B (C) is such that

$$\frac{2\pi}{N_s} \cdot p_1 \cdot x = K \cdot \frac{2\pi}{3} \quad ; \quad K - \text{integer} \neq 3^p \quad (7.22)$$

Replacing  $N_s$  from Equation 7.21 and Equation 7.22 yields the following:

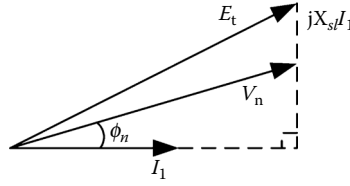
$$\frac{\pi}{3} \cdot \left( \frac{d}{bd+c} \right) \cdot x = K \cdot \frac{2\pi}{3} \quad ; \quad x = 2 \cdot \frac{bd+c}{d} \cdot K \quad (7.23)$$

Now,  $x$  has to be an integer, and  $2K$  has to be divisible by  $d$ . Also,  $d$  may not contain a  $3^p$  factor, as this is eliminated from  $K$ . For fractionary windings, not only should  $N_s$  be a multiple of three, but also the denominator  $c$  of  $q$  should not contain three as a factor. According to Equation 7.21, if  $N_s$  contains a factor of  $3^p$ , then  $p_1$  (pole pairs number) should also contain it, so that it would not appear in  $c$ .

In large SGs, the stator core is made of segments (Chapter 4) because the size of the lamination sheets is limited to 1 to 1.1 m in width. The number of slots per segment  $N_{ss}$ , for  $N_c$  segments, is

$$N_{ss} = \frac{N_s}{N_c} \quad (7.24)$$

For details on stator core segments, revisit Chapter 4. In general, it is advisable that  $N_{ss}$  be an even number, so that  $N_s$  has to be an even number. But in such cases, apparently, only integer  $q$  values are feasible. For fractionary windings,  $N_{ss}$  may be an odd number and contain three as a factor. Moreover, large stator bore diameter hydrogenerators have their stator cores made of a few  $N_k$  sections that are wound at the



**FIGURE 7.8** The total electromagnetic field (emf)  $E_t$ .

manufacturer's site and assembled at the user's site. So, the number of slots  $N_s$  has to be divisible by both  $N_c$  and  $N_k$ .

In large SGs, the stator coil turns are made by transposed copper bars, and generally, there is one turn (bar) per coil. So, the total number of turns for all three phases ( $3 \cdot a \cdot W_a$ ) is equal to the number of slots  $N_s$ :

$$N_s = 3 \cdot a \cdot W_a \quad (7.25)$$

where

$a$  = the number of current paths in parallel

$W_a$  = the turns per path/phase

On the other hand, the number of turns  $W_a$  per current path is related to the flux per pole and the resultant emf  $E_t$  per phase:

$$E_t = \pi \sqrt{2} \cdot f_n \cdot (W_a \cdot K_{W1}) \cdot \Phi_1 \quad (7.26)$$

$$\Phi_1 = \frac{2}{\pi} \cdot B_{g1} \cdot \tau \cdot l_{Fe} \quad ; \quad \tau \approx \pi D / 2 p_1 \quad (7.27)$$

with  $\tau$  equal to the pole pitch of stator winding. At this stage,  $l_{Fe} \approx l_p$ ,  $D$ ,  $\tau$  and are known, and  $B_{g1}$  is the airgap rated flux density that is chosen in the interval given in the previous paragraph. The winding factor  $K_{W1}$  is

$$K_{W1} = K_{q1} K_{y1} \quad (7.28)$$

$$K_{q1} \approx \frac{\sin \pi / 6}{q \sin \pi / 6 q} \quad ; \quad K_{y1} = \sin \frac{y}{\tau} \frac{\pi}{2} \quad (7.29)$$

with  $y/\tau$  equal to the coil span/pole pitch. For fractionary  $q = (bd + c)/d$ ,  $q$  will be replaced in Equation 7.29 by  $bd + c$ .

From Figure 7.8, the emf (airgap emf) is

$$E_t \approx V_n \sqrt{1 + x_{sl} (2 \sin \phi_n + x_{sl})} \approx (1.07 - 1.1) \cdot V_n \quad (7.30)$$

The leakage reactance  $x_{sl}$  is generally less than 0.15 or

$$x_{sl} \approx x'_{sl} \times (0.35 - 0.4) \quad (7.31)$$

This value of  $x_{sl}$  is only orientative and will be recalculated later in the design process.

$V_n$  root mean squared (RMS) is the rated phase voltage of the SG. The rated current  $I_n$  is as follows:

$$I_n = \frac{P_n}{3V_n \cos \phi_n} \quad (7.32)$$

with  $\phi_n$  equal to the rated power factor angle (specified).

The number of current paths in parallel depends on many factors, such as type of winding (lap or wave), number of stator sectors, and so forth.

With tentative values for  $a$  and  $W_a$  found from (Equation 7.26 through Equation 7.30) and  $W_a$  rounded to a multiple of three (for three phases), the number of slots is calculated from Equation 7.25. Then, it is checked if  $N_s$  is divisible by the number of stator sections  $N_K$ . The number of stator sections is

$$\begin{aligned} N_K &= 2 \text{ for } D < 4 \text{ m} \\ N_K &= 4 \text{ for } D = 4 \div 8 \text{ m} \\ N_K &= 6 \text{ (8) for } D > 8 \text{ m} \end{aligned} \quad (7.33)$$

To yield a symmetrical winding for fractionary  $q = b + c/d$ , we need  $2p_1/d = \text{integer}$  and, as pointed out above,  $d/3 \neq \text{integer}$ . With  $a$  current paths,

$$d \leq \frac{2p_1}{a} \quad (7.34)$$

It is also appropriate to have a large value for  $d$ , so that the distribution factor of higher space harmonics be small, even if, by necessity,  $c/d = 1/2$ ,  $b > 3$ .

For wave windings, the simplest configuration is obtained for

$$\frac{3c \pm 1}{d} = \text{integer} \quad (7.35)$$

So, the best  $c/d$  ratios are as follows:

$$\frac{c}{d} = \frac{2}{5}, \frac{3}{5}, \frac{2}{7}, \frac{5}{7}, \frac{3}{8}, \frac{5}{8}, \frac{3}{10}, \frac{7}{10}, \frac{4}{11}, \frac{7}{11}, \frac{4}{13}, \frac{9}{13} \dots \quad (7.36)$$

For  $d = 5, 7, 11, 13, \dots$ :

$$\frac{6c \pm 1}{d} = \text{integer} \quad (7.37)$$

with

$$\frac{c}{d} = \frac{1}{5}, \frac{4}{5}, \frac{1}{7}, \frac{6}{7}, \frac{2}{11}, \frac{9}{11}, \frac{2}{13}, \frac{11}{13} \dots \quad (7.38)$$

A low level of noise with fractionary windings requires

$$3 \cdot \left( b + \frac{c}{d} \right) \pm \frac{1}{d} \neq \text{integer} \quad (7.39)$$

With  $c/d = 1/2$  ( $b > 3$ ), the subharmonics are cancelled.



More details on choosing the number of slots for hydrogenerators can be found in Reference [6].

## 7.5 Design of Stator Winding

The main stator winding types for SGs were introduced in [Chapter 4](#). For turbogenerators, with  $q > 4$  (5), and integer  $q$ , two-layer windings with lap or wave-chorded coils are typical. They are fully symmetric with  $60^\circ$  phase spread per each pole.

### Example 7.1: Integer $q$ Turbogenerator Winding

Take a numerical example of a two-pole turbogenerator with an interior stator diameter  $D_{is} = 1.0$  m and with a typical slot pitch  $\tau_s \approx 60$  to 70 mm. Find an appropriate number of slots for integer  $q$ , and then build a two-layer winding for it.

The number of slots  $N_s$  is

$$N_s = 2p \cdot q \cdot m \quad ; \quad N_s \cdot \tau_s = \pi \cdot D_{is} \quad (7.40)$$

So,

$$q = \text{integer} \left( \pi \cdot \frac{D_{is}}{\tau_s} \cdot \frac{1}{2p_1 \cdot m} \right) = \text{integer} \left( \frac{\pi \cdot 1.0}{0.07} \cdot \frac{1}{2 \cdot 1 \cdot 3} \right) \quad (7.41)$$

where

$$\begin{aligned} \tau_s &= 0.0654 \text{ m} \\ q &= 8 \end{aligned}$$

With a stator stack length  $l_{Fe} = 4.5$  m,  $B_{g1} = 0.837$  T,  $V_A = 12/\sqrt{3}$  kV,  $f_n = 60$  Hz,  $E_t = 1.10 V_n$  (Equation 7.30), and  $a = 2$  current paths, the number of turns per current path/phase,  $W_a$ , is as follows (Equation 7.26):

$$W_a = \frac{E_t}{\pi \sqrt{2} \cdot f_n \cdot K_{w1} \cdot \Phi_1} \quad (7.42)$$

$$\Phi_1 = \frac{2}{\pi} \cdot B_{g1} \cdot \tau \cdot l_{Fe} = \frac{2}{\pi} \cdot 0.837 \cdot \frac{\pi}{2} \cdot 1 \cdot 4.5 = 3.7674 \text{ Wb/pole} \quad (7.43)$$

$$K_{w1} = \frac{\sin \pi/6}{8 \cdot \sin(\pi/(6 \cdot 8))} \times \sin \frac{21}{24} \cdot \frac{\pi}{2} = 0.9556 \times 0.966 = 0.9230 \quad (7.44)$$

So, from Equation 7.42,

$$W_a = \frac{1.07 \times (12/\sqrt{3}) \cdot 10^3}{\pi \sqrt{2} \times 60 \times 0.923 \times 3.7674} = 8.012 \approx 8 = q \quad (7.45)$$

Fortunately, the number of turns per current path, which occupies just one pole of the two, is equal to the value of  $q$ . A multiple of  $q$  would also be possible. With  $W_a = 8$ , we have one turn/coil, so the coils are made of single bars aggregated from transposed conductors.

From Equation 7.25, the number of stator slots  $N_s$  is

$$N_s = a \times W_a \times 3 = 2 \times 8 \times 3 = 48 \quad (7.46)$$

The condition  $W_a = q$  (or  $kq$ ) could be fulfilled with modified stator bore diameter or stack length or slot pitch.

In small machines,  $W_a = kq$  with  $k > 2$ .

Building an integer  $q$  two-layer winding comprises the following steps:

- The electrical angle of emfs in two adjacent slots  $\alpha_{es}$ :

$$\alpha_{es} = \frac{2\pi}{N_s} \cdot p_1 = \frac{2\pi}{48} \cdot 1 = \frac{\pi}{24} \quad (7.47)$$

- The number  $t$  of slots with in-phase emfs:

$$t = \text{largest common divisor } (N_s, p_1) = p_1 = 1 \quad (7.48)$$

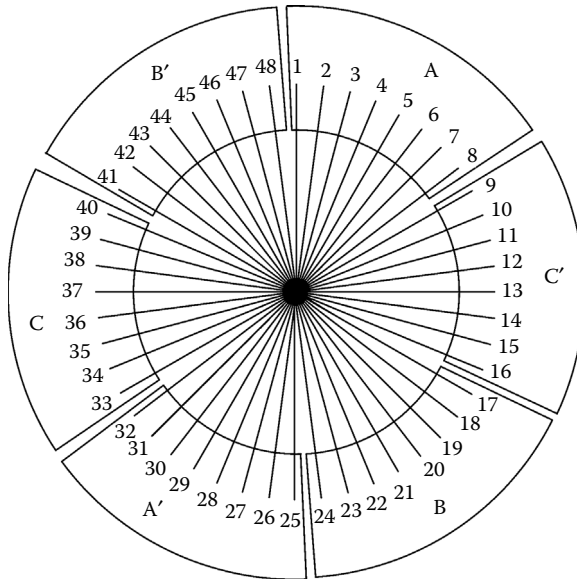
- The number of distinct slot emfs:

$$N_s/t = 48/1 = 48 \quad (7.49)$$

- The angle of neighboring distinct emfs:

$$\alpha_{ef} = \frac{2\pi \cdot t}{N_s} = \frac{2\pi \cdot 1}{48} = \frac{\pi}{24} = \alpha_{es} \quad (7.50)$$

- Draw the star of slot emfs with  $N_s/t = 48$  elements (Figure 7.9).



**FIGURE 7.9** Electromagnetic field (emf) star for  $2p_1 = 2$  and  $N_s = 48$ .

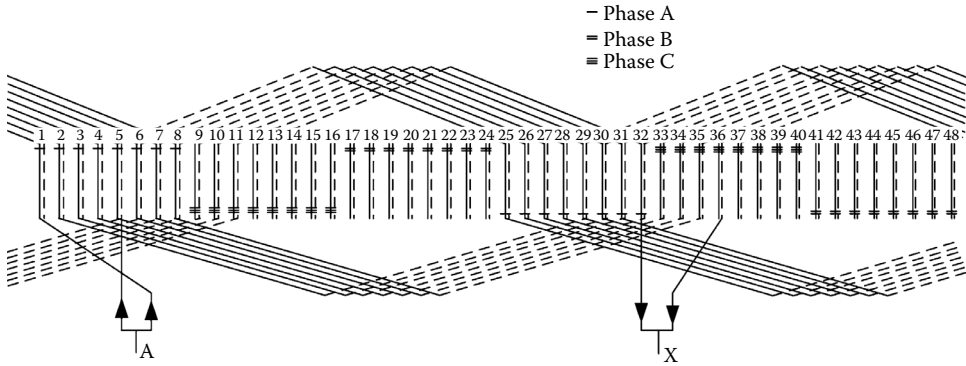


FIGURE 7.10 Two-pole, bar-wave winding with  $N_s = 48$  slots,  $q = 8$  slot/pole/phase, and  $y/\tau = 20/24$ .

- Divide the distinct emfs in  $2 \times m = 6$  equal zones. Opposite zones represent the in and out slots of a phase in the first layer. The angle between the beginnings of phases A, B, C is  $\frac{2\pi}{3}$ , clockwise.
- From each in-and-out slot phase, coils are initiated in layer one and completed in layer two, from left to right, according to the coil span  $y$ :  $y = \frac{20}{24}\tau = 20$  slot pitches (Figure 7.10).

Making use of bar-wave coils and two current paths in parallel, practically no additional connectors are necessary to complete the phase.

The single-turn bar coils with wave connections are usually used for hydrogenerators ( $2p_1 > 4$ ) to reduce overall connector length — at the price of some additional labor. Here, the very large power of the SG at only 12 kV line voltage imposed a single-turn coil winding.

Doubling the line voltage to 24 kV would lead to a two-turn coil winding, where lap coils are generally preferable.

For the fractionary windings, so typical for hydrogenerators, after setting the most appropriate value of fractionary  $q$  in the previous paragraph, an example is worked out here.

### Example 7.2: The Fractionary $q$ Winding

Consider the case of a 100 MVA hydrogenerator designed at  $V_{nl} = 15,500$  V (RMS line voltage, star connection),  $f_n = 50$  Hz,  $\cos\phi_n = 0.9$ ,  $n_n = 150$  rpm, and  $n_{max} = 250$  rpm.

Calculate the main stator geometry and then with  $B_{gl} = 0.9$  T, the number of turns with one current path, and design a single-turn (bar) coil winding with fractionary  $q$ .

#### Solution

For indirect air cooling (see Figure 7.7) with the power per pole,

$$\frac{S_n}{2p_1} = \frac{100 \cdot 10^6}{40} = 2.5 \cdot 10^3 \text{ kVAmin/pole} \quad (7.51)$$

the output power coefficient  $C = 9 \text{ kVAmin/m}^3$ .

The maximum rotor diameter (Equation 7.12) for  $U_{max} = 140 \text{ m/sec}$ :

$$D = \frac{U_{\max}}{\pi \cdot n_{\max}} = \frac{140 \text{ m/sec}}{\pi \cdot (250/60) \text{ rad/sec}} = 10.70 \text{ m} \quad (7.52)$$

The ideal stack length  $l_i$  is calculated from Equation 7.9:

$$l_i \approx \frac{S_n}{C \cdot D^2 \cdot n_n} = \frac{100000(kVA)}{9 \left( \frac{kVA_{\min}}{m^3} \right) (10.70)^2 (m^2) 150(rpm)} = 0.647 \text{ m} \quad (7.53)$$

The flux per pole  $\Phi_1$  (Equation 7.8) is

$$\Phi_1 = \frac{2}{\pi} \cdot B_{g1} \cdot \tau \cdot l_i = \frac{2}{\pi} \cdot 0.9 \cdot \frac{\pi \cdot 10.7}{40} \cdot 0.647 = 0.3115 \text{ Wb/pole} \quad (7.54)$$

With  $E_{t1}/V_{nph} = 1.07$  and an assumed winding factor  $K_{W1} \approx 0.925$ , the number of turns per current path  $W_a$  is as follows (Equation 7.26):

$$W_a = \frac{1.1 \cdot V_{nph}}{\pi \sqrt{2} \cdot f_n \cdot K_{W1} \cdot \Phi_1} = \frac{1.07 \times 15500 / \sqrt{3}}{\pi \sqrt{2} \cdot 50 \cdot 0.925 \cdot 0.3115} \approx 150 \text{ turns/path/phase} \quad (7.55)$$

For one current path, the total number of turns for all three phases (equal to the number of slots  $N_s$ ) would be

$$N_s = 3 \cdot a \cdot W_a = 3 \cdot 1 \cdot 150 = 450 \text{ slots} \quad (7.56)$$

A tentative value of  $q_{ave}$  would be

$$q_{ave} = \frac{N_s}{2 p_1 \cdot m} = \frac{450}{40 \cdot 3} = \frac{450}{120} = 3 \frac{3}{4} \quad (7.57)$$

It is obvious that this value of  $q$  is not among those suggested in Equation 7.37 and Equation 7.38, but Equation 7.35 is half fulfilled, as  $c = 3$ ,  $d = 4$ , and

$$\frac{3c-1}{d} = \frac{3 \cdot 3-1}{4} = 2 = \text{integer} \quad (7.58)$$

With 15 slots per segment ( $N_{ss} = 15$ ), the total number of segments  $N_c$  per section is

$$N_c = \frac{N_s}{N_{ss} \cdot N_K} = \frac{450}{15 \times 6} = 5 \quad (7.59)$$

So, the total number of segments in the stator core is  $N_c \cdot N_K = 5 \cdot 6 = 30$ . For a 10.7 m rotor diameter, this is a reasonable value (lamination sheet width is less than 1.1 to 1.2 m).

Though  $N_{ss} = 15$  slots/segment is an odd (instead of even) number, it is acceptable.

Finally, we adopt  $q_{ave} = 3 \frac{3}{4}$  for a 40-pole single-turn bar winding with one current path.

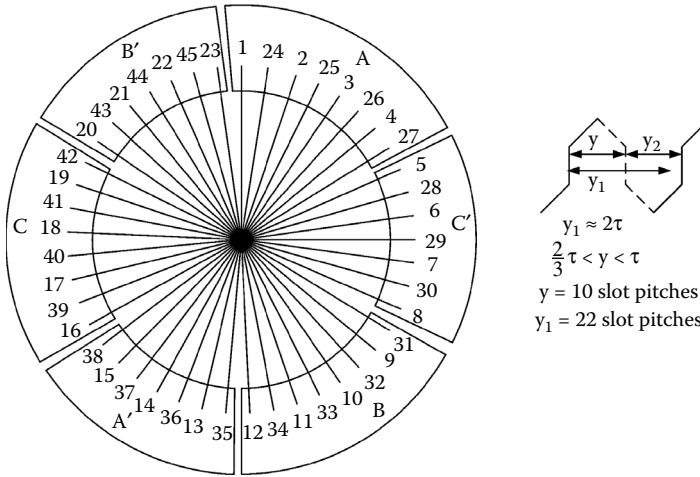


FIGURE 7.11 The electromagnetic force (emf) star for  $N_s = 450$  slots,  $2p_1 = 40$  poles for the first distinct 45 slots.

To build the winding, we adopt a similar path as for integer  $q$ :

- Calculate the slot emf angle:  $\alpha_{ec} = \frac{2\pi}{N_s} \cdot p_1 = \frac{2\pi}{450} \cdot 20 = \frac{4\pi}{45}$
- Calculate the highest common divisor  $t$  of  $N_s$  and  $p_1$ :  $t = 10 = p_1/2$
- Find the number of distinct slot emfs:  $N_s/t = 450/10 = 45$
- Find the angle between neighboring distinct emfs:  $\alpha_{et} = \frac{2\pi \cdot t}{N_s} = \frac{2\pi \cdot 10}{450} = \frac{2\pi}{45} = \frac{\alpha_{ec}}{2}$
- Draw the emf star, observing that only 45 of them are distinct, and every ten of them overlap each other (Figure 7.11)

As there are only 45 ( $N_s/t$ ) distinct emfs, it is enough to consider them alone, as the situation repeats itself identically ten times. After four poles ( $d = 4$  in  $q = b + c/d$ ), the situation repeats.

- Calculate  $\frac{N_s/t}{3} = \frac{450/10}{3} = 15$ , and start by allocating phase A — eight in emfs (slots) and seven out emfs — such that the eight and the seven are in phase opposition as much as possible. In our case, in slots for phase A are (1, 2, 3, 4, 24, 25, 26, 27), and out slots are (13, 14, 15, 35, 36, 37, 38).
- Proceed the same way for phases B and C by allowing groups of eight and seven neighboring slots to alternate. The sequence (clockwise) is A, C', B, A', C, B' to complete the circle.
- The division of slots between the two layers is valid in layer 1; for layer 2, the allocation comes naturally by observing the coil span  $y$ :

$$y \leq \text{integer} \left( \frac{N_s}{2p_1} \right) = \text{integer} \left( \frac{450}{40} \right) = 11 \text{ slot pitches} \quad (7.60)$$

It is possible to choose  $y = 9, 10, 11$ , but  $y = 10$  seems a good compromise in reducing the fifth and seventh space harmonics while not reducing the emf fundamental too much.

Note that for fractionary  $q$ , some of the connections between successive bars of a bar-wave winding have to be made of separate (nonwave) connectors.

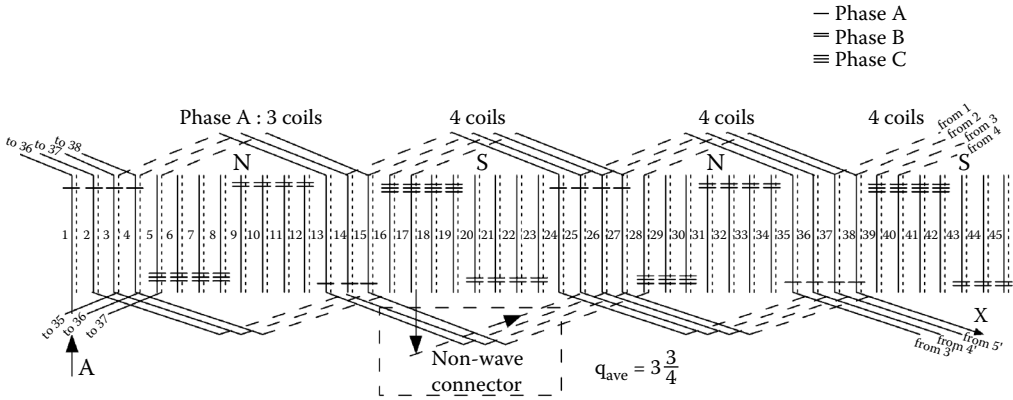


FIGURE 7.12 450-slot, 40-poles,  $q = 3 \frac{3}{4}$  bar-wave winding/phase A for the first  $N_s/t = 45$  slots.

For a minimum number of such additional connectors,  $y_1$  (Figure 7.11) should be as close as possible to two times the pole pitch, and  $6q$  equals the integer.

In our case,  $6q = 6 \cdot \frac{15}{4} = \frac{45}{2} \neq \text{integer}$ , so the situation is not ideal. But the symmetry of the winding is notable, as there are ten identical zones of the winding, each spanning four poles.

An example of the first 45 slots with all phase allocation completed, but only with phase A coils shown, is given in Figure 7.12.

As Figure 7.12 shows, there is only one nonwave connector per phase for  $N_s/t$  section of machine.

A simple rule for allocation of slots per phases is apparent from Figure 7.12.

Based on the sequence A, C', B, A', C, B', ..., we allocate for each phase group  $d$  slots for  $b$  groups and then  $c$  slots for one group and repeat this sequence for all the slots of the machine. Again,  $d$  should not be divisible by three for symmetry ( $q = b + c/d$ ).

The allocation of slots to phase may also be done through tables [6], but the principle is the same as above.

The emf star has the added advantage of allowing for simple verifications for phase balance by finding the position of the resultant emf of each phase after adding the up (forward) and the opposite of down (backward) emfs.

It is also evident that the distribution factor formula (Equation 7.29) may be adopted for the purpose by noting that the number of vectors included should not be  $q$  but the denominator of  $q$ , that is,  $bd + c$ , in our case,  $3 \cdot 3 + 3 = 15 = 8 + 7$  vectors:

$$K_{q1} = \frac{\sin \frac{\pi}{6}}{(bd + c) \cdot \sin \left( \frac{\pi}{6(bd + c)} \right)} \quad (7.61)$$

The chording factor  $K_{y1}$  formula (Equation 7.29) still holds.

## 7.6 Design of Stator Core

By now, in our design process, the rotor diameter  $D$ , the stator core ideal length  $l_p$ , the pole pitch  $\tau$ , and the number of slots  $N_s$  are already calculated as shown in previous paragraphs.

To design the stator core, the stator bore diameter  $D_{is}$  is first required. But to accomplish this, the airgap  $g$  has to be calculated first, because

$$D_{is} = D + 2g \quad (7.62)$$

Calculating or choosing the airgap should account for the following:

- Required SCR (or  $1/x_d$ )
- Reduced airgap flux density harmonics due to slot openings so as to limit the emf time harmonics within standards requirements
- Increased excitation winding losses with larger airgap
- Reduced stator space harmonics losses in the rotor with larger airgap, for a given stator slotting
- Varied mechanical limitation on airgap during operation by at most 10% of its rated value

The trend today is to impose smaller SCR (0.4 to 0.6), that is, smaller airgap, to reduce the excitation winding losses. Transient stability is to be preserved through fast exciter voltage forcing by adequate control. With smaller airgap, care must be exercised in estimating the emf time harmonics and the additional rotor surface (or cage) losses.

So, it seems reasonable to adopt the airgap based on a preliminary calculated value of  $x_d$ :

$$x_d = x_{sl} + x_{ad} \quad (7.63)$$

At this point,  $x_{sl}$  may be assigned a value  $x_{sl} = 0.1 - 0.15$  or  $x_{sl} \approx (0.35 - 0.4)x'_d$ , when  $(x'_d)_{\max}$  is imposed as a specification.

The magnetization reactance  $X_{ad}$  (in  $\Omega$ ) is

$$X_{ad} = K_{ad} \frac{6\mu_0\omega_1 (W_a \cdot K_{W1})^2 \cdot \tau \cdot l_i}{\pi^2 \cdot g \cdot K_C \cdot (1 + K_{sl})} = x_{ad} \left( \frac{V_n}{I_n} \right)_{\text{phase}} \quad (7.64)$$

$$(I_n)_{\text{phase}} = \frac{S_n}{3(V_n)_{\text{phase}} \cos\phi_n} \quad (7.65)$$

$K_{ad}$  is a reduction coefficient of  $d$  axis magnetizing reactance when a salient pole rotor is used ([Chapter 4](#)):

$$K_{ad} \approx \frac{\tau_p}{\tau} + \frac{1}{\pi} \sin\left(\frac{\tau_p}{\tau} \cdot \pi\right) \quad ; \quad \tau_p - \text{rotor pole shoe span} \quad (7.66)$$

Equation 7.66 is valid for constant airgap salient poles. For hydrogenerators, with reduced airgap, the airgap under the salient poles varies to yield a more sinusoidal airgap flux density distribution. With  $\tau_p/\tau \approx 0.62 - 0.75$ , in general,  $K_{ad} > 0.9$  for uniform airgap, but it is lower for increased nonuniform airgap.

The Carter coefficient,  $K_C$ , which includes the influence of slot openings and the effect of radial channels in the stator core stack, is also unknown at this stage of the design but, typically,  $K_C < 1.15$ .

Finally, the magnetic saturation level is not known yet, but it is known to be less than 0.25 ( $K_{sd} < 0.25$ ). Basically, Equation 7.64 and Equation 7.65, with assigned values of  $K_{ad}$ ,  $K_c$ , and  $K_{sd}$  and known winding data  $W_a$ ,  $K_{wl}$  (from the previous paragraph), provide a preliminary value for the airgap to secure the required value of  $x_d$ .

A traditional expression for airgap is as follows:

$$g = 4.0 \cdot 10^{-7} \frac{A \cdot \tau}{(x_d - 0.1) B_{g1}} \quad (7.67)$$

where

$A$  = the linear current loading (A/m)

$B_{g1}$  = the design airgap flux density (specified)

$\tau$  = the pole pitch  $\tau \approx \pi D / 2p_1$

$x_{sl} = 0.1$  is the assigned value of stator leakage reactance in P.U.

Knowing the rated current  $I_n$  and the number of current path  $a$  (current loading),  $A$ , is as follows:

$$A \approx \frac{6 \cdot W_a \cdot a \cdot I_{na}}{\pi \cdot D} \quad (7.68)$$

For  $SCR = 0.5$ ,  $x_d = 2$ , and  $D = 10.7$  m,  $a = 1$ ,  $W_a = 150$ ,  $I_{na} = 4000$  A (Example 7.2),  $2p_1 = 40$  poles,  $B_{g1} = 0.9$  T,

$$g = 4.0 \times 10^{-7} \cdot \frac{6 \cdot W_a \cdot a \cdot I_{na}}{(x_d - 0.1) \cdot 2p_1 \cdot B_{g1}} = \frac{4 \cdot 10^{-7} \times 6 \times 150 \times 4000}{40 \cdot 0.9 \cdot (2 - 0.1)} = 0.02105 \text{ m} \quad (7.69)$$

Now the pole pitch  $\tau$  is

$$\tau = \frac{\pi(D + 2g)}{2p_1} = \frac{\pi(10.7 + 2 \cdot 0.02105)}{40} = 0.8435 \text{ m} \quad (7.70)$$

Note that the rather small specified SCR led to a high  $x_d$  ( $x_d = 2.0$ ) and, thus, to a rather small airgap for this 10.7 m rotor diameter with a 0.8435 m pole pitch.

Turbogenerators are characterized by a larger airgap for the same  $A$ ,  $B_{g1}$ , and SCR, as  $\tau$  is notably larger. Moreover, the smaller periphery length (smaller diameter) in turbogenerators imposes larger values of  $A$  than in hydrogenerators — one more reason for a larger airgap. Airgaps of 60 to 70 mm in two-pole, 1.2 m rotor diameter turbogenerators are not uncommon. This preliminary airgap value is to be modified if the desired  $x_d$  is not obtained, or some of the mechanical, emf harmonics or additional losses constraints are not met.

The stator terminal line voltage is chosen based on the following:

- Insulation costs
- Insulation maintenance costs
- Step-up transformer, power switches, and protection costs

Generally, the higher the power, the higher the voltage. Also, the voltage is higher for direct-cooled windings, because the transmission through the conductor and slot insulation to the slot walls is no longer a main constraint.



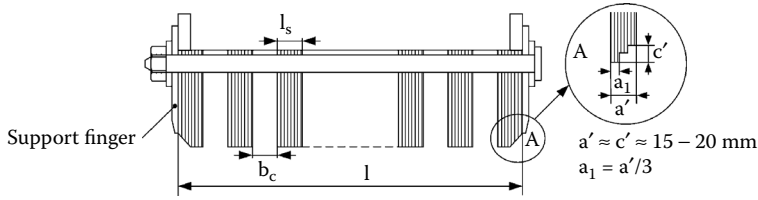


FIGURE 7.13 Stator core with radial channels.

As a starting point,

$$V_{nl} \approx 6 - 7 \text{ kV for } S_n < 20 \text{ MVA}$$

$$10 - 11 \text{ kV for } S_n \approx (20 - 60) \text{ MVA}$$

$$13 - 14 \text{ kV for } S_n \approx (60 - 75) \text{ MVA} \quad (7.71)$$

$$15 - 16 \text{ kV for } S_n \approx (175 - 300) \text{ MVA}$$

$$16 - 28 \text{ kV for } S_n > 300 \text{ MVA}$$

Recently, 56 kV and 100 kV cable-winding SGs were proposed.

### 7.6.1 Stator Stack Geometry

As radial or radial-axial cooling is used (Figure 7.13), there are  $n_c$  radial channels, and each cooling channel is  $b_c$  wide. The total iron length  $l_i$  is as follows:

$$l_i = l - n_c b_c \quad (7.72)$$

The ideal length  $l_i$  is approximately

$$l_i = \left[ l - n_c \cdot b'_c - 2a' \cdot \left( 1 - \frac{g}{g + c'/2} \right) \right] \cdot k_{Fe} \quad (7.73)$$

with  $b'_c$  an equivalent cooling channel width that is smaller than  $b_c$  and dependent on airgap  $g$ . The larger the airgap, the smaller will be  $b'_c/b_c$ . Generally,  $b_c = 8$  to  $12$  mm, and the elementary stack width  $l_s = 45$  to  $60$  mm. When the airgap  $g$  is larger than  $b_c$ ,  $b'_c/b_c < 0.2 - 0.3$  due to the large fringing flux.  $K_{Fe}$  is the iron filling factor that accounts for the existing insulation layer between laminations. For 0.5 mm thick laminations,  $K_{Fe} \approx 0.93$  to  $0.95$ .

The open stator slots may house uni-turn (bar) coils (Figure 7.14a) or multiturn (two, in general) coils (Figure 7.14b) placed in two layers. The single- and two-turn coils are made of multiple rectangular cross-sectional conductors in parallel that have to be fully transposed (Figure 7.15a and Figure 7.15b) in large power SGs (Roebel bars). Typically, the elementary conductor height  $h_c$  is less than 2.5 mm.

The elementary conductors are transposed to cancel eddy currents induced by each of them in the others, thus reducing drastically the total skin effect AC resistance factor. (More details are presented in the forthcoming paragraph on stator resistance.) The transposition provides for positioning each elementary conductor in all the positions of the other conductors, along the stack length. The transposition step along stack length is above 30 mm, and there should be as many transpositions as there are elementary conductors used to make a turn.

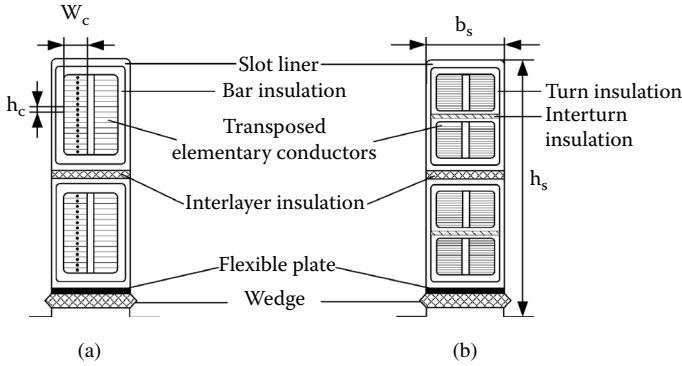


FIGURE 7.14 Stator conductors in slot (indirect cooling): (a) single-turn bar winding and (b) two-turn coil winding.

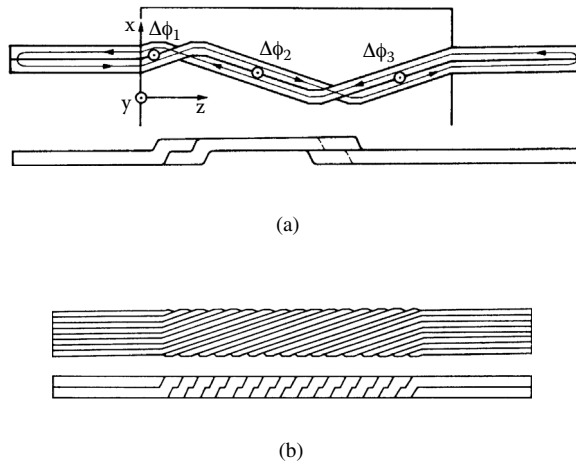


FIGURE 7.15 Roebel bar: (a) two conductors and (b) complete Roebel bar.

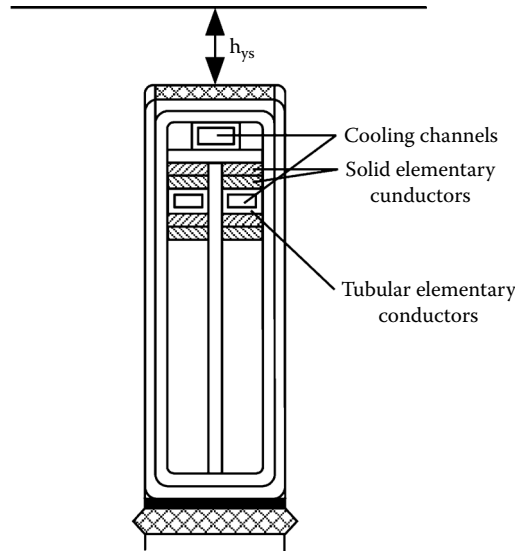
The thickness of various insulation layers depends on the terminal voltage and on the number of slots. Generally,

$$\begin{aligned} \frac{2W_c}{b_s} &= \frac{\text{copper width}}{\text{slot width}} \approx 0.6-0.7 \\ \frac{b_s}{\tau_s} &= \frac{\text{slot width}}{\text{slot pitch}} \approx 0.35-0.55 \\ \tau_s &= \pi \cdot D_{is} / 6 p_1 q_1 \\ \frac{h_s}{b_s} &= \frac{\text{slot height}}{\text{slot width}} = 4-10 \end{aligned} \quad (7.74)$$

For direct cooling, the copper area per slot area is smaller than that for indirect cooling, because each elementary conductor has an interior channel for the coolant.

A single-layer single turn per coil winding, as shown in Figure 7.16, exhibits sequences of two solid elementary conductors followed by a tubular conductor.

It is also possible to use only tubular elementary conductors.



**FIGURE 7.16** Single-layer winding with direct cooling.

The slot area  $A_{slotu}$  may be calculated by knowing the total current per slot, the design current density  $j_{cos}$ , and the total copper filling factor  $K_{fill}$ :

$$A_{slotu} = \frac{6 \cdot W_a \cdot I_n}{N_s \cdot K_{fill} \cdot j_{cos}} ; A_{slot} = b_s \cdot h_s \quad (7.75)$$

The output power coefficient secures values of ampere turns per slot that lead to fulfilling constraints (Equation 7.74).

The design current density depends on the adopted cooling system, and for start values, Table 7.1 may be used. It should also be noticed that the terminal voltage impresses lower limits on slot width with orientative values from 15 mm below 6 kV (line voltage) up to 35 to 40 mm at 24 kV. The slot total filling factor goes down from values of up to 0.55 below 6 kV to less than 0.3 to 0.35 at 20 kV and higher, for indirect cooling. Smaller values of  $K_{fill}$  are practical for direct cooling windings.

With stator bore diameter  $D_{is}$ , number of stator slots  $N_s$ , rated path current  $I_n$ , number of turns per current paths in parallel  $W_a$ , already assigned  $K_{fill}$ ,  $j_{cos}$ , from Equation 7.75 with Equation 7.74, the rectangular stator slot may be sized by calculating  $h_s$  and  $b_s$ . Finally, all insulation layers are accounted for, and a more exact filling factor is obtained.

The stator yoke height  $h_{ys}$  is simply

$$h_{ys} \approx \frac{B_{g1}}{B_{y1}} \cdot \frac{\tau}{\pi} ; B_{y1} = 1.3 - 1.7 \text{ T} \quad (7.76)$$

where

$\tau$  = the stator pole pitch

$B_{y1}$  = the design stator yoke flux density

As the slots are rectangular, the teeth are rather trapezoidal, so the tooth flux density  $B_{t1}$  varies along the radial direction. The maximum value  $B_{tmax}$  occurs approximately at the slot top:

$$B_{t\max} \approx B_{g1} \cdot \frac{\tau_s}{\tau_s - b_s} \approx 1.6 - 2.0 \text{ T} \quad (7.77)$$

In Equation 7.77, the reduction of tooth flux density due to the fringing flux lines through the slots is neglected.

### Example 7.3: Stator Slot and Yoke Sizing

For the same hydrogenerator as discussed in Example 7.2, with  $S_n = 100$  MVA,  $I_n = 4000$  A,  $U_{nl} = 15$  kV,  $2p_1 = 40$ ,  $D = 10.7$  m,  $l_i = 0.647$  m,  $N_s = 450$ , airgap  $g = 2.1 \times 10^{-2}$  m,  $B_{g1} = 0.9$  T,  $W_a = 150$  turns/current path, and  $a = 1$  current paths, determine (for indirect air cooling), size of the stator slot and yoke, and the stator core outer diameter  $D_{os}$ .

#### Solution

For indirect air cooling, a total slot filling factor is adopted  $K_{fill} = 0.4$ .

The current density (Table 7.1) is  $j_{cos} = 6.0$  A/mm<sup>2</sup>.

From Equation 7.75, the slot useful area  $A_{slotu}$  is as follows:

$$A_{slotu} = \frac{6 \cdot W_a \cdot I_n}{N_s \cdot K_{fill} \cdot j_{cos}} = \frac{6 \cdot 150 \cdot 4000}{450 \cdot 0.4 \cdot 6 \cdot 10^6} = 3333 \cdot 10^{-6} \text{ m}^2$$

The slot pitch  $\tau_s$  is

$$\tau_s = \frac{\pi(D + 2g)}{N_s} = \frac{\pi(10.7 + 2 \cdot 0.021)}{450} = 74.95 \cdot 10^{-3} \text{ m}$$

The slot width is selected according to Equation 7.74:

$$b_s = \tau_s \cdot 0.4 = 74.95 \cdot 10^{-3} \approx 30 \cdot 10^{-3} \text{ m}$$

The maximum tooth flux density is as follows:

$$B_{t\max} = B_{g1} \cdot \frac{\tau_s}{\tau_s - b_s} = 0.9 \cdot \frac{74.95}{74.95 - 30} = 1.5 \text{ T}$$

The slot height  $h_s$  may now determined from Equation 7.75:

$$h_s \approx \frac{A_{slotu}}{b_s} = \frac{3333 \cdot 10^{-6}}{30 \times 10^{-3}} = 111 \cdot 10^{-3} \text{ m}$$

The ratio  $h_s/b_s = 111/30 = 3.7033 < 4$ , as suggested in Equation 7.74.

The rather low  $h_s/b_s$  ratio tends to produce a low stator slot leakage inductance, that is also a reduction in  $x'_d$ . As the maximum value of  $x'_d$  is limited for transient stability reasons, it may be adequate to retain this slot geometry.

The moderate  $B_{t\max}$  does not account for further reduction of the tooth width in the wedge area.

With stator yoke flux density  $B_{ys} = 1.4$  T, the stator yoke height  $h_{ys}$  (Equation 7.76) is

$$h_{ys} = \frac{B_{gl}}{B_{ys}} \cdot \frac{\tau}{\pi} = \frac{0.9}{1.4} \cdot 74.95 \times 10^{-3} \cdot \frac{450}{40} \cdot \frac{1}{\pi} = 172.74 \times 10^{-3} \text{ m}$$

The external stator diameter is

$$D_{os} = D + 2g + 2h_s + 2h_{ys} = 10.7 + 2 \cdot 0.0201 + (2 \times 111 + 2 \times 172) \cdot 10^{-3} = 11.309 \text{ m}$$

In general, the stator yoke height  $h_{ys}$  should be larger than the slot height  $h_s$  to avoid large noise and vibration at  $2f_n$  frequency.

## 7.7 Salient-Pole Rotor Design

Hydrogenerators and most industrial generators make use of salient-pole rotors. They are also found in some wind generators above 2 MW/unit.

The airgap under the rotor pole shoe gets larger toward the pole shoe ends (Figure 7.17).

In general,  $g_{\max}/g = 1.5$  to  $2.5$  to make the airgap flux density, produced by the field current, sinusoidal. Ideally,

$$g(\theta) = \frac{g}{\cos(\theta p_1)} \quad (7.78)$$

In reality, the pole shoe may be cut from 1 to 1.8 mm laminations along a circle with radius  $R_p < D/2$ , where  $D$  is the rotor diameter at minimum airgap ( $g$ ).

The pole shoe  $b_p$  per pole pitch  $\tau$  is a compromise between leaving enough room for field coils and limiting the interpole flux linkage:

$$\alpha_i = \frac{b_p}{\tau} \approx 0.66 - 0.75 \quad (7.79)$$

Generally,  $\alpha_i$  increases with the pole pitch  $\tau$ , reaching 0.75 at  $\tau = 0.8$  m. Also, the ratio between the pole shoe span  $b_p$  and the stator slot pitch  $\tau_s$  should be  $b_p/\tau_s > 5.5$  to avoid notable pulsations in the emf due to stator slotting. With  $q \geq 3$ , this condition is met automatically for all values of  $\alpha_i$  in Equation 7.79.

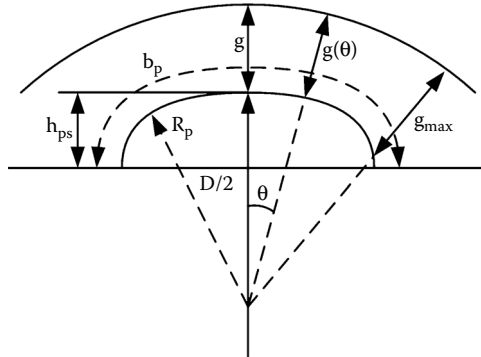


FIGURE 7.17 Variable airgap salient pole.

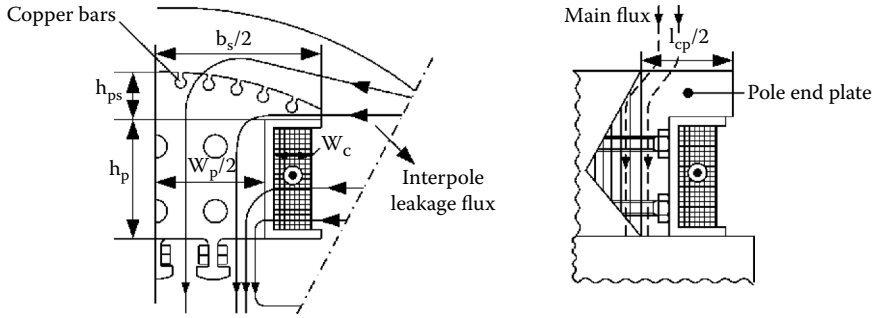


FIGURE 7.18 Salient rotor pole construction.

Given the central and maximum airgaps ( $g$  and  $g_{max}$ ), rotor diameter  $D$ , and the pole shoe span  $b_p$ , the radius  $R_p$  of the pole shoe shape is approximately

$$R_p = \frac{D}{2} \cdot \frac{1}{\left(1 + \frac{4 \cdot D \cdot (g_{max} - g)}{b_p^2}\right)} < \frac{D}{2} \quad (7.80)$$

The cross-section through a salient rotor pole is shown in Figure 7.18.

The length of pole body (made of 1 to 2 mm thick die-cast laminations)  $l_p$  is made smaller than stator core total length  $l$  by around 50 to 80 mm, while the end plates (Figure 7.18),  $l_{ep}$ , made of solid iron, are  $l_{ep} = 50$  to 120 mm.

So, the total ideal length of rotor pole  $l_{pi}$  is as follows:

$$l_{pi} = l - (0.05 + 0.08) + l_{ep} > l \quad (7.81)$$

The effective iron length of rotor  $l_{pFe}$  is

$$l_{pFe} \approx l_{pi} \cdot K_{Fe} \quad (7.82)$$

The lamination filling factor (due to insulation layers)  $K_{Fe} \approx 0.95$  to  $0.97$  for lamination thickness going from 1 to 1.8 mm. The total length of rotor  $l_{pi}$  is still larger than the stator stack length  $l$  in order to further reduce the flux density in the rotor pole body with width  $W_p$  (Figure 7.18) that is, in general,

$$W_p \approx (0.45 - 0.55) \cdot \tau \quad (7.83)$$

The wound rotor pole height  $h_p$  per pole  $\tau$  pitch ratio  $K_h$  decreases with the pole pitch  $\tau$  and with increased average airgap flux density:

$$K_h = \frac{h_p}{\tau} \quad (7.84)$$

In general, for  $B_{ga} = 0.7$  T ( $B_{gl} = 0.9$  T),  $K_h$  starts from 0.3 at  $\tau = 0.4$  m and ends at 0.1 for  $\tau = 1$  m. Higher values of  $K_h$  may be used for smaller airgap flux densities.

To design the field winding, the rated,  $V_{fn}$ , and peak,  $V_{fmax}$ , voltages have to be known, together with field pole mmf  $W_f I_f$ . By  $I_{fn}$ , we mean the excitation current required to produce full voltage at full load and rated power factor. At this stage of the design method,  $I_{fn}$  is not known, and it may not be calculated

rigorously, because the rotor pole and yoke design is not finished. But, a preliminary design of rotor pole and yoke is feasible here.

#### Example 7.4: Salient-Pole Rotor Preliminary Design

For the data in Example 7.3, let us design the salient-pole rotor. The ratio  $g_{\max}/g = 2.5$ .

##### Solution

Knowing the pole pitch and choosing a conservative  $\alpha_i = 0.7$ , from Equation 7.79, the pole width  $b_p$  is as follows (Example 7.3):

$$b_p \approx \alpha_i \cdot \tau = 0.7 \cdot \frac{\pi(10.7 + 2 \cdot 0.021)}{40} = 0.7 \cdot 0.843 = 0.59 \text{ m}$$

The radius of rotor pole shoe  $R_p$  (Equation 7.80) is

$$R_p = \frac{D}{2} \cdot \frac{1}{\left(1 + \frac{4 \cdot D \cdot (g_{\max} - g)}{b_p^2}\right)} = \frac{10.7}{2} \cdot \frac{1}{1 + \frac{4 \cdot 10.7}{0.59^2} \cdot (2.5 - 1) \cdot 2.1 \cdot 10^{-2}} = 1.0978 \text{ m}$$

The rotor pole shoe height at center  $h_{ps}$  (Figure 7.17) should be large enough to accommodate the damper winding and is proportional to the pole pitch:

$$\frac{h_{ps}}{\tau} = \approx 0.1 \text{ for } \tau = 0.3 \text{ m}$$

$$\approx 0.2 \text{ for } \tau = 1 \text{ m}$$

The pole body width  $W_p$  is chosen from Equation 7.83:

$$W_p = 0.5 \cdot \tau = 0.5 \cdot 0.843 = 0.4215 \text{ m}$$

Consequently, the space left for coil width  $W_c$  is

$$W_c = \frac{b_p - W_p}{2} = \frac{0.59 - 0.4215}{2} = 0.08425 \text{ m}$$

The pole body (and coil) height  $h_p = K_h \cdot \tau = 0.18 \cdot 0.843 = 0.1517 \text{ m}$ .

So, with a total coil filling  $K_{\text{fill}} = 0.62$  design current density  $j_{\text{cor}} = 10 \text{ A/mm}^2$ , the ampere-turns of field coil per pole are as follows:

$$W_F I_{Fn} = j_{\text{cor}} \cdot W_c \cdot h_p \cdot K_{\text{fill}} = 10 \cdot 151.7 \cdot 84.25 \cdot 0.62 = 79240 \text{ At}$$

On the other hand, the stator rated mmf per pole  $F_{ln}$  is

$$F_{ln} = \frac{3\sqrt{2} \cdot W_a \cdot K_{w1} \cdot I_n}{\pi \cdot p_1} = \frac{3\sqrt{2} \cdot 150 \cdot 0.925 \cdot 4000}{\pi \cdot 20} = 37383 \text{ At/pole}$$

As

$$\left(F_n/W_F I_{Fn}\right)^{-1} = \frac{79240}{37383} = 2.12$$

there are chances that the calculated rated field pole mmf  $W_F I_{Fn}$  will suffice for rated power, rated voltage, and rated power factor.

However, also notice that the rated current density was raised to 10 A/mm<sup>2</sup> in the rotor, in contrast to 6 A/mm<sup>2</sup> in the stator. The much shorter end connections justify this choice. Later in the design, the exact  $W_F I_{Fn}$  value will be calculated.

The rotor yoke design is basically similar to the stator yoke design, but there is an additional, leakage (interpole) magnetic flux to consider. Later, it will be calculated in detail, but for now, a 10 to 15% increase in polar flux is enough to allow for preliminary calculation of the rotor yoke radial height  $h_{yr}$ :

$$h_{yr} \approx \frac{B_{g1}}{B_{yr}} \cdot \frac{\tau}{\pi} (1 + K_{peak}) = \frac{0.9}{1.5} \cdot \frac{0.843}{\pi} \cdot (1 + 0.12) = 0.1804 \text{ m}$$

This is a conservative value.

Though the design methodology can produce a detailed analytical calculation of no-load and on-load magnetization curves, only with the finite element method (FEM) can we provide exact distributions of flux density in the various parts of the machine for given operating conditions.

## 7.8 Damper Cage Design

Stator space mmf harmonics of order 5, 7, 11, 13, 17, 19, ..., as well as airgap permeance harmonics due to slot openings, induce voltages and thus produce currents in the rotor damper winding. These stator mmf aggregated space harmonics are reduced drastically by fractionary windings ( $q = b + c/d$ ), with first slot harmonics that is  $6(bd + c) \pm 1$ . When  $bd + c > 9$ , these harmonics are negligible; thus, it is feasible to use the same slot pitch in the stator  $\tau_s$  and in the rotor  $\tau_d: \tau_s = \tau_d$ . However, for integer  $q$  or  $bd + c < 9$  or  $q = b + 1/2$ :

$$\tau_s \neq \tau_d \quad (7.85)$$

Otherwise, the induced currents in the damper windings by the stator slotting harmonics are augmented when  $\tau_s = \tau_r$ .

For these cases, it is recommended [7] that

$$\begin{aligned} \tau_r &< \tau_s \\ \frac{2\tau}{\tau_r} &\geq 6q \pm 1 + c \quad ; \quad c = 0 \text{ for } q = \text{integer} \\ b_p - 2\tau_r &= \frac{2k\tau}{6q + 1} \end{aligned} \quad (7.86)$$

In Reference [6], the condition  $N/p_1 = 2K_1 \pm 1/2$  is demonstrated to lead to the reduction of bar-to-bar currents due to the first slot opening harmonic of the stator. But, the second slot opening harmonic ( $v = 2$ ) may violate this condition.



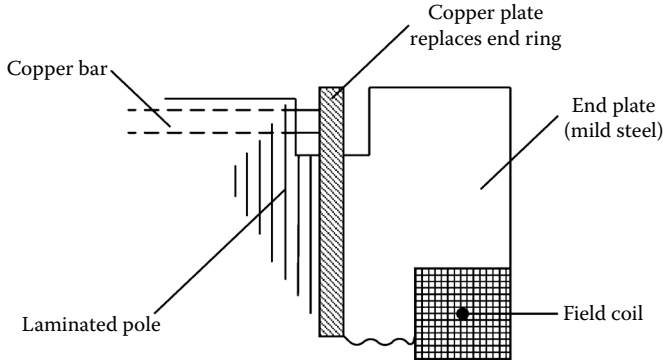


FIGURE 7.19 Copper end plate replaces end ring.

The number of damper bars per pole  $N_2$  is as follows:

$$N_2 \approx \frac{b_p}{\tau_r} - 1 \quad (7.87)$$

In some cases, the damper cage may be left out, but then the pole shoe (at least) should be made of solid mild steel.

The cross-section of the damper cage bars per pole represents a fraction of stator slot area per pole:

$$A_{bar} = \frac{1}{N_2} \cdot (0.15 - 0.3) \cdot \frac{6 \cdot W_a \cdot I_n}{2 p_1 \cdot j_{COs}} \quad (7.88)$$

The cage bars are round and made of copper or brass, so their diameters  $d_{bar}$  are standardized:

$$d_{bar} = \sqrt{\frac{4}{\pi} \cdot A_{bar}} \quad (7.89)$$

The cage bars are connected through partial or integral end rings. The cross-section of end ring  $A_{ring}$  is about half the cross-sectional area of all bars under a pole:

$$A_{ring} \approx (0.5 - 0.6) \cdot N_2 \cdot A_{bar} \quad (7.90)$$

The complete end rings, though useful in providing  $x_d'' \approx x_q''$  and  $q$  axis current damping during transients, hamper the free axial circulation of cooling agent between rotor poles. Thus, it is practical to use copper end plates that follow the shape of the poles and extend below the first row of pole bolts. They are located between the laminated rotor pole core and the end plate made of steel (Figure 7.19). For good contact with the copper bars, the copper end plate should have a thickness of about 10 mm [6]. The copper plate plays the role of the complete end ring but without obstructing the cooler axial flow between the rotor poles. Also, it is mechanically more rugged than the latter.

## 7.9 Design of Cylindrical Rotors

The cylindrical rotor is generally made from solid iron with milled slots over about two thirds of periphery so as to produce  $2p_1$  poles with distributed field coils in slots (Chapter 4 and Figure 7.20). Slots are radial

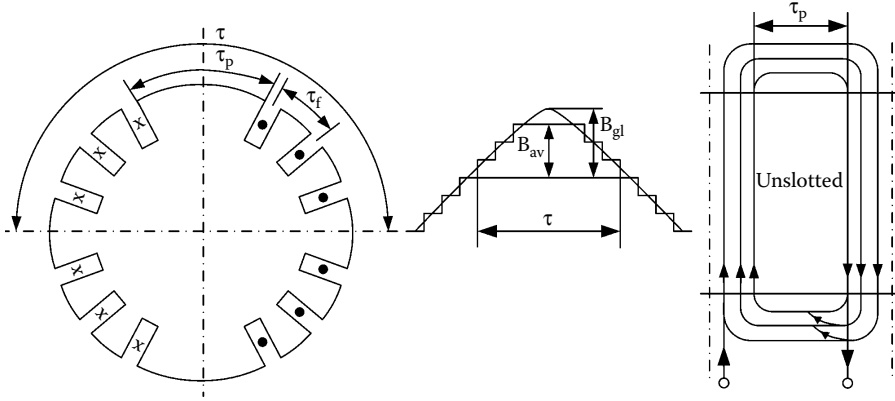


FIGURE 7.20 Two-pole cylindrical rotor with field coils.

and open in Figure 7.20. According to Chapter 4, Equation 4.23, the airgap flux density produced by the distributed field winding is

$$B_{gv}(x) = K_{fv} \cdot B_{gav} \cdot \cos v \frac{\pi}{\tau} x \quad (7.91)$$

in rotor coordinates, with

$$K_{fv} \cong \frac{8}{v^2 \pi^2} \cdot \frac{\cos v \frac{\tau_p}{\tau} \cdot \frac{\pi}{2}}{1 - v \tau_p / \tau} \quad (7.92)$$

Only odd harmonics are present if all poles are balanced. For the fundamental, the form factor  $K_{fv}$  is as follows:

$$(K_{f1})_{\frac{\tau_p}{\tau} = \frac{2}{3}} = \frac{8}{\pi^2} \cdot \frac{\cos \frac{\tau_p}{\tau} \cdot \frac{\pi}{2}}{1 - \tau_p / \tau} = \frac{8}{\pi^2} \cdot \frac{\sqrt{3}/2}{1 - 1/3} = 1.0528 \quad (7.93)$$

The third harmonic is already reduced:

$$(K_{f3})_{\frac{\tau_p}{\tau} = \frac{2}{3}} = \frac{8}{9 \cdot \pi^2} \cdot \frac{\cos 3 \frac{\tau_p}{\tau} \cdot \frac{\pi}{2}}{1 - 3 \tau_p / \tau} = -0.1415 \quad (7.94)$$

The stator and rotor slot opening airgap permeance influence on the excitation airgap flux density harmonics is not considered in Equation 7.92.

The rotor excitation slot pitch  $\tau_f$  should be chosen in relation to stator slot pitch  $\tau_s$ , such that the stator emf harmonics and solid rotor eddy current losses are minimized. Further,

$$B_{av} = \frac{\mu_0 \cdot W_{fc} \cdot I_{fa} \times (N_{fp} / 2)}{g \cdot K_c \cdot (1 + K_s)} \quad (7.95)$$

$N_{fp}$  is the number of field-winding slots per rotor pole.  $K_c$  is the Carter coefficient accounting for the apparent increase of airgap due to stator and rotor slotting and for the presence of radial cooling channels (if any). Magnetic saturation is accounted for by  $K_s$ , with  $K_s < 0.2 \div 0.25$ .

### Example 7.5

Consider a two-pole 30 MVA, 50 Hz,  $\cos\phi_n = 0.9$  lagging turbogenerator. With a stator bore diameter  $D_{is} = 0.85$  m,  $N_{fp(\text{in the rotor})} = 12$  slots/pole,  $B_{gl} = 0.825$  T,  $A = 56,000$  A/m,  $SCR = 0.55$ ,  $V_{fn} = 500$  V, and  $V_{fmax} = 2 V_{fn}$ .

Design the pertinent field winding after calculating the necessary airgap  $g$ .

### Solution

The ampere-turns per meter  $A$  may be turned into mmf per pole  $F_{ln}$ :

$$\tau = \pi \cdot D_{is} / 2 = \pi \cdot 0.83 / 2 = 1.303 \text{ m}$$

$$F_{ln} \approx \frac{A \cdot \tau}{2} = \frac{56000 \cdot 1.303}{2} = 36349 \text{ At/pole}$$

The no-load equivalent field winding mmf fundamental per pole  $F_{f10}$  is

$$F_{f10} \approx SCR \cdot F_{ln} = 0.5 \times 36349 = 18242 \text{ At/pole}$$

With  $K_s = 0.25$ ,  $K_c = 1.1$ , and  $N_{fp} = 12$ , the airgap flux density produced at no load by the field winding is

$$B_{g10} = \frac{\mu_0 \cdot F_{f10}}{g \cdot K_c \cdot (1 + K_s)} \quad (7.96)$$

So, the airgap  $g$  becomes

$$g = \frac{1.256 \cdot 10^{-6} \cdot 18242}{1.1 \cdot 1.25 \cdot 0.825} = 20.2 \times 10^{-3} \text{ m}$$

Consequently, from Equation 7.89 with Equation 7.85 and Equation 7.87, the rotor pole slot mmf  $W_{fc} I_{fo}$  at no load is as follows:

$$I_{fo} W_{fc} = \frac{2}{N_{fp}} \cdot \frac{F_{f10}}{K_{f1}} = \frac{2}{12} \cdot \frac{18242}{1.0528} = 2887.8 \text{ At/slot} \quad (7.97)$$

At full load and rated power factor, the excitation mmf requirement is about two times larger than that for no load:

$$W_{fc} \cdot I_{fn} = 2 W_{fc} \cdot I_{fo} = 5775.7 \text{ At/slot} \quad (7.98)$$

The slot pitch of rotor slots  $\tau_r$  is

$$\tau_{fr} = \frac{\pi(D_{is} - 2g) \cdot (1 - \tau_p / \tau)}{2p_1 \cdot N_{fp}} = \frac{\pi(0.83 - 2 \cdot 0.02) \cdot (1 - 0.3)}{2 \cdot 12} = 0.07235 \text{ m}$$

The value of  $\tau_p / \tau = 0.3$  is taken to avoid  $\tau_{fr} = \tau_s$  (slot pitch in the stator with  $q = 6$ ).

With the slot fill factor  $K_{fill} = 0.5$  (profiled conductors), and a design current density  $j_{cor} = 6 \text{ A/mm}^2$ , the rotor slot useful area  $A_{slotr}$  is as follows:

$$A_{slotr} = \frac{W_{fc} I_{fn}}{j_{cor} \cdot k_{fill}} = \frac{5775.7}{6 \cdot 0.5} = 1925.23 \text{ mm}^2$$

The slot width  $W_{sr}$  is

$$W_{sr} = 0.4 \cdot \tau_{fr} = 0.4 \times 0.07235 = 0.02894 \approx 0.029 \text{ m}$$

The slot useful height  $h_{sr}$  is

$$h_{sr} = \frac{A_{slotr}}{W_{sr}} = \frac{1925.23 \cdot 10^{-6}}{0.029} = 66.38 \cdot 10^{-3} \text{ m}$$

The aspect ratio of the slot is rather small ( $h_{sr}/W_{sr} = 2.289$ ); therefore, the current density might be reduced or, if needed, higher field mmfs than in Equation 7.98 are feasible.

To finish the design, calculate the number of turns per coil and the conductor cross-section.

The field-circuit rated voltage  $V_{fn}$  should be considered when designing the field winding, with the voltage ceiling left for field current forcing during transients to enhance transient stability limits with a small SCR = 0.5.

First, the field-winding resistance per pole  $R_{fp}$  has to be calculated:

$$R_{fp} = \rho_{co} \frac{N_{fp} \cdot l_{ave}}{I_{fn}} \cdot W_{fc} \cdot a_p \cdot j_{cor} \quad (7.99)$$

where  $l_{ave}$  is the average length of turn. Approximately,

$$l_{ave} \approx 2 \cdot l_{pi} + \pi \cdot K_{av} \cdot \frac{\tau}{2} \quad ; \quad K_{av} \approx 0.5 - 0.7 \quad (7.100)$$

Considering  $a_p$  current paths in the rotor, the field voltage equation under steady state is

$$V_{fn} = R_f \cdot I_{fn} \quad (7.101)$$

with

$$R_f = \frac{R_{fp} \times 2p_1}{a_p^2} \quad (7.102)$$

Making use of Equation 7.99 and Equation 7.100 in Equation 7.101 yields the following:

$$V_{fn} = \rho_{co} \cdot \frac{N_{fp} \cdot l_{ave} \cdot j_{cor}}{W_{fc} \cdot (I_{fn}/a_p)} \cdot W_{fc}^2 \times \frac{2p_1}{a_p^2} \cdot I_{fn} = \rho_{co} \cdot N_{fp} \cdot l_{ave} \cdot j_{cor} \cdot \frac{2p_1}{a_p} \cdot W_{fc} \quad (7.103)$$

Equation 7.103 provides for the direct computation of the number of turns per field coil. The copper resistivity should be considered at rated temperature.

### Example 7.6: Field Coil Sizing

Calculate, for the rotor in Example 7.5, the number of turns per field coil and the wire cross-section if the stator core total length  $l$  is 2.5 m. Also,  $I_{fn}W_{fc} = 5775$  At/coil.

#### Solution

The turn average length is as follows (Equation 7.100):

$$l_{avef} = 2 \cdot \left( 2.6 + \pi \cdot 0.6 \cdot \frac{1.303}{2} \right) \cong 7.65 \text{ m}$$

With  $\rho_{co} = 2.15 \times 10^{-8} \Omega\text{m}$ ,  $j_{cor} = 6 \text{ A/mm}^2$ ,  $a_p = 2$  current paths in parallel,  $2p_1 = 2$ ,  $N_{fp} = 12$  slots/rotor pole, and  $V_{fn} = 500 \text{ V}$ , from Equation 7.103, the number of turns per field coil (same for all)  $W_{fc}$  is

$$W_{fc} = \frac{V_{fn} \cdot a_p}{\rho_{co} \cdot N_{fp} \cdot l_{ave} \cdot j_{cor} \cdot 2p_1} = \frac{500 \cdot 2}{2.15 \times 10^{-8} \cdot 12 \cdot 7.65 \cdot 6 \times 10^6 \cdot 2 \cdot 1} = 42.22 \text{ turn/coil}$$

Let us adopt  $W_{fc} = 42$  turns/coil.

The total field current  $I_{fn}$  comes from the known  $I_{fn}W_{fc}$ :

$$I_{fn} = \frac{W_{fc} \cdot I_{fn}}{W_{fc}} = \frac{5755}{42} = 137.50 \text{ A}$$

The current per path (in the coils)  $I_{fna}$  is

$$I_{fna} = \frac{I_{fn}}{a_p} = \frac{137.5}{2} = 68.75 \text{ A}$$

The copper conductor cross-section  $A_{co}$  is

$$A_{co} = \frac{I_{fna}}{j_{cor}} = \frac{68.75}{6 \times 10^6} = 11.458 \cdot 10^{-6} \text{ m}^2$$

A single rectangular cross-section wire may be used.

The total rated power in the excitation winding  $P_{exn}$  is

$$P_{exn} = V_{fn} \cdot I_{fn} = 500 \times 137.5 = 68750 \text{ W} \quad (7.104)$$

For a 30 MW SG, this means only 0.229%.

The rather small airgap ( $g = 20 \times 10^{-3}$  m), the moderate rated current density ( $j_{cor} = 6 \times 10^6$  A/m<sup>2</sup>), and the 2/1 ratio between full load and no-load field mmf may justify the rather small power (0.229%) in the field winding.

## 7.10 The Open-Circuit Saturation Curve

The open-circuit saturation curve basically represents the no-load generator phase voltage  $E_{10}$  as a function of excitation current (or mmf)  $I_f$ , at rated frequency:

$$E_{10} = \sqrt{2} \cdot f_n \cdot W_a \cdot K_{W1} \cdot \frac{2}{\pi} \cdot B_{g1} \cdot \tau \cdot l_i = K_E \cdot \Phi_{1g} \quad (7.105)$$

Also at no load, from Equation 7.27, Equation 7.91, and Equation 7.97,

$$\Phi_{1g} = \frac{2}{\pi} \cdot l_i \cdot \tau \cdot B_{g1} ; B_{g1} = K_{f1} \cdot B_{av} ; B_{av} = \frac{\mu_0 (W_f I_f)_{pole}}{g_a K_c (1 + K_s)} \quad (7.106)$$

The saturation factor  $K_s$  depends on  $I_F$ , that is, on  $B_{av}$  and the machine stator and rotor core geometry and the  $B(H)$  curves of stator and rotor core materials. The form factor  $K_{f1}$  is as follows (Chapter 4):

$$K_{f1} \approx \frac{4}{\pi} \cdot \sin \frac{\tau_p}{\tau} \cdot \frac{\pi}{2} \quad \text{for salient rotor poles}$$

$$K_{f1} \approx \frac{8}{\pi^2} \cdot \frac{\cos \frac{\tau_p}{\tau} \cdot \frac{\pi}{2}}{1 - \frac{\tau_p}{\tau} \cdot \pi} \quad \text{for cylindrical rotor poles} \quad (7.107)$$

The equivalent stator stack iron length  $l_i$  is as follows (Equation 7.73):

$$l_i \approx \left( l - n_c \cdot b'_c - 2a' \cdot \left( 1 - \frac{g}{g + c'/2} \right) \right) \quad (7.108)$$

The Carter coefficient  $K_C$  is, in general, the product of at least two of three terms:

- $K_{C1}$  — accounting for airgap increase due to stator slot openings
- $K_{C2}$  — accounting for airgap increase due to rotor slotting (caused for damper cage slots or by the field-winding slots)
- $K_{C3}$  — accounting for the airgap increase due to radial channels opening  $b_c$

When the airgap varies under the salient rotor pole shoe from  $g$  to  $g_{\max}$ , in calculating  $K_{C1}$ ,  $K_{C2}$ , and  $K_{C3}$ , an average airgap  $g_a$  is used:

$$g_a \approx \frac{2}{3} g + \frac{1}{3} g_{\max} \quad (7.109)$$

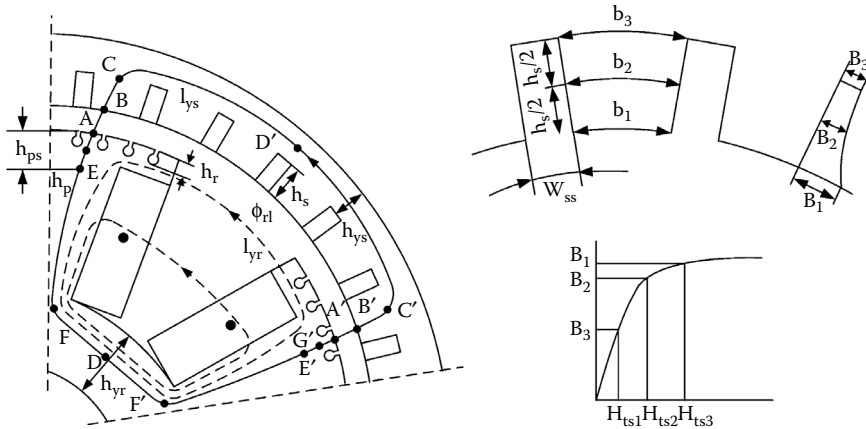


FIGURE 7.21 Average no-load flux path in the synchronous generator.

The literature on induction machines abounds with analytical formulas for Carter coefficients. A simplified practical version is given here:

$$K_{C_1} = \frac{\tau_s + 10g_a}{(\tau_s - W_{ss}) + 10g_a} \quad ; \quad W_{ss} - \text{slot opening, } \tau_s - \text{stator slot pitch}$$

$$K_{C_2} = \frac{\tau_r + 10g_a}{(\tau_r - W_{rr}) + 10g_a} \quad ; \quad \tau_r - \text{rotor slotting pitch, } W_{rr} - \text{rotor slot opening} \quad (7.110)$$

$$K_{C_3} = \frac{\tau_c + 10g_a}{(\tau_c - \tau_b) + 10g_a} \quad ; \quad \tau_c - \text{radial channel average pitch, } b_c - \text{radial channel width}$$

$$K_C = K_{C_1} \cdot K_{C_2} \cdot (K_{C_3})^i \quad (7.111)$$

The value of  $i$  is  $i = 1$  if the radial channels are present only in the stator, but it is  $i = 2$  when they are present in both the stator and rotor.

When the airgap is constant (cylindrical rotors),  $g_a = g$ , as expected.

We will proceed to an analytical calculation of the open-circuit magnetization curve, because we previously defined all the components for given airgap flux density  $B_{g1}$  (or  $B_{av}$ ). Except for one — the interpole rotor leakage flux,  $\Phi_{rl}$ , which is dependent on the mmf drop along the airgap + stator teeth + yoke:  $F_{rl} = F_{AA'}$  (Figure 7.21).

According to Ampere's law, for the average flux line,

$$(F_{10})_{pole} = F_g + F_{ts} + F_{ys} + F_{tr} + F_{ps} + F_p + F_{yr} \quad (7.112)$$

AB BC CD AG GE EF FD'

Also,

$$F_{rl} = 2 \cdot (F_g + F_{ts} + F_{ys}) = F_{AA'} \quad (7.113)$$

$$F_{10} = F_{rl} + F_{rr} \quad ; \quad F_{rr} = 2(F_{tr} + F_{ps} + F_p + F_{yr})$$

with

$$\begin{aligned}
 F_g &= \frac{B_{g1}}{\mu_0} \cdot g_a \cdot K_C \\
 F_{ts} &= H_{ts}^{av} \cdot h_s \\
 F_{ys} &= H_{ys}^{av} \cdot l_{ys}^{av} \\
 F_{tr} &= H_{tr}^{av} \cdot h_r \\
 F_{ps} &= H_{ps}^{av} \cdot h_{ps} \\
 F_p &= H_p^{av} \cdot h_p \\
 F_{yr} &= H_{yr}^{av} \cdot l_{yr}^{av}
 \end{aligned} \tag{7.114}$$

For given  $B_{g1}$  and machine geometry, the flux densities in various stator regions may be calculated.

The average value of field  $H_{ts}^{av}$  is calculated as follows for the trapezoidal stator teeth:

$$H_{ts}^{av} = (H_{ts1} + 4H_{ts2} + H_{ts3})/6 \tag{7.115}$$

$H_{ts1}$ ,  $H_{ts2}$ , and  $H_{ts3}$  correspond to the tooth flux densities  $B_1$ ,  $B_2$ , and  $B_3$  in the three locations indicated in [Figure 7.21](#):

$$\begin{aligned}
 B_1 &\approx B_{g1} \cdot \frac{\tau_s}{b_1} \quad ; \quad b_1 = \tau_s - W_{ss} \\
 B_2 &\approx B_1 \cdot \frac{b_1}{b_2} \\
 B_3 &\approx B_1 \cdot \frac{b_1}{b_3}
 \end{aligned} \tag{7.116}$$

The widths of the stator teeth  $b_1$ ,  $b_2$ , and  $b_3$  at tooth top, middle, and bottom, respectively, are straight-forward. From the known lamination magnetization curves,  $H_{ts1}$ ,  $H_{ts2}$ , and  $H_{ts3}$ , corresponding to  $B_1$ ,  $B_2$ , and  $B_3$ , are obtained. A similar procedure may be used to calculate the mmf drop  $F_{tr}$  in the rotor tooth. For the stator yoke, the maximum value of the flux density is used to obtain  $H_{ys}$  from the same magnetization curve:

$$B_{ys}^{\max} \approx \frac{B_{g1}}{h_{ys}} \cdot \frac{\tau}{\pi} \tag{7.117}$$

However, to account for the fact that lower flux density levels exist in the yoke and the lengths of various flux lines in this zone are different from each other, flux line length  $l_{ys}^{av}$  is to be defined:

$$l_{ys}^{av} \approx K_{ys} \frac{\pi(D + 2g + 2h_s + h_{ys})}{4p_1} \quad ; \quad K_{ys} \approx (0.66 - 0.8) \tag{7.118}$$

Also,



$$l_{ys}^{av} \approx \pi(D - 2h_{ps} - 2h_p) / 4p_1 \quad (7.119)$$

Realistic values of  $K_{ys}$  may be obtained through FEM or multiple magnetic circuit field distribution calculation methods [8,9].

The total pole flux in the rotor also includes the interpole leakage flux  $\Phi_{rl}$  besides the airgap flux  $\Phi_{lg}$ :

$$\Phi_{lg} = \frac{2}{\pi} \cdot B_{g1} \cdot \tau \cdot l_i \quad (7.120)$$

$$\Phi_{lr} = \Phi_{lg} + 2\Phi_{rl}$$

Recognize that not all the sections of the rotor pole encounter the entire rotor leakage flux  $\Phi_{rl}$ , but the rotor yoke does. When calculating the dependence of leakage rotor flux  $\Phi_{rl}$  on the mmf  $1/2 F_{AB} = F_{rl}$  (Equation 7.113), either analytical or numerical flux distribution investigation is necessary.

However, as the tangential distance between neighboring rotor poles in air is notable, to a first approximation, we have

$$\Phi_{rl} = P_{rl} \cdot F_{rl} \quad (7.121)$$

There are a few analytical approximations for  $P_r$  (the permeance of the leakage interpolar flux) [6, 7]. Here, we use the similitude of the interpolar space with a semiclosed slot plus the airgap flux permeance known as zigzag (airgap) leakage [10]:

$$P_{rl} = 2\mu_0 \cdot (\lambda_p + \lambda_f) \cdot l_{pi} \quad (7.122)$$

$$\lambda_p \approx \frac{h_{ps1}}{(b_{r1} + b_{r2})} + \frac{1}{3} \frac{h_{ps2}}{(b_{r2} + b_{r3})} + \frac{1}{3} \frac{h_{p1}}{(b_{r3} + b_{r4})} \quad (7.123)$$

$$\lambda_f = \frac{5g_a K_c / 2b_{r1}}{5 + (4g_a K_c / 2b_{r1})} \quad (7.124)$$

Once the geometry of the rotor is known, all variables in Equation 7.123 and Equation 7.124 are given, and with  $F_{rl}$  — the corresponding mmf (Equation 7.113) — also calculated, the interpolar leakage flux is obtained.

Now for the rotor pole shoe, pole body, rotor yoke average flux density  $B_{ps}^{av}$ ,  $B_p^{av}$ ,  $B_{yr}$  calculations, the leakage flux has to be added to the airgap flux per pole:

$$\begin{aligned} B_{ps}^{av} &\approx \frac{1/\pi \cdot B_{g1} \cdot \tau \cdot l_i + C_{ps} \cdot \Phi_{rl}}{l_{pi} \cdot (b_p/2)} \quad ; \quad C_{ps} = 0.3 - 0.5 \\ B_p^{av} &\approx \frac{1/\pi \cdot B_{g1} \cdot \tau \cdot l_i + C_p \cdot \Phi_{rl}}{l_{pi} \cdot (W_p/2)} \quad ; \quad C_p \approx 0.75 - 0.85 \\ B_{yr} &\approx \frac{1/\pi \cdot B_{g1} \cdot \tau \cdot l_i + \Phi_{rl}}{l_{pi} \cdot h_{yr}} \end{aligned} \quad (7.125)$$



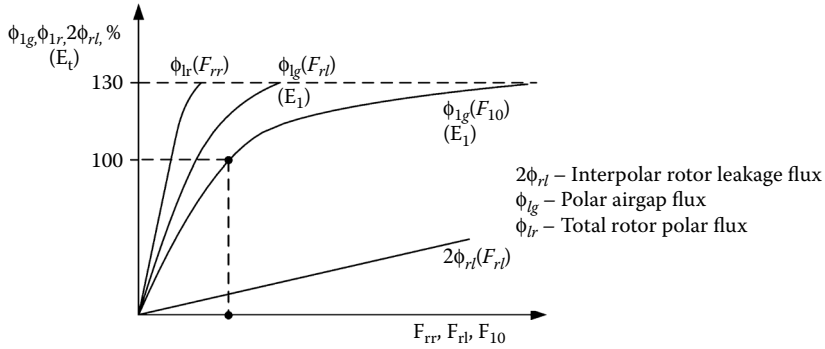


FIGURE 7.23 Partial no-load magnetization curves.

The airgap flux  $\Phi_{1g}$  is proportional to emf  $E_1$  (Equation 7.105). So, in P.U. values  $\Phi_{1g}$  and  $E_1$  are superimposed in Figure 7.23.

## 7.11 The On-Load Excitation mmf $F_{1n}$

Calculating the on-load excitation  $F_{1n}$  per pole is essential in designing the SG in relation to field-winding losses and overtemperatures.

Traditionally, there were two methods used to calculate  $F_{1n}$ :

- Potier diagram method
- Partial magnetization curve method

The Potier diagram is meant for cylindrical rotors, while the partial magnetization curve method is necessary for the salient-pole rotor. What is needed in both methods is the rated armature mmf  $F_{a1}$ , the rated voltage, and the leakage stator reactance  $x_d$ . At this stage in the design,  $x_d$  may be calculated.

For now, we consider it known (in general,  $x_d = 0.09$  to  $0.15$  P.U. for all SGs and increases with power). The Potier reactance is as follows:

$$x_p \approx x_d + 0.02 \text{ P.U.} \quad (7.129)$$

### 7.11.1 Potier Diagram Method

The diagram in Figure 7.24 is drawn in P.U. with rated terminal voltage  $V_n$  as the base voltage. Also, the base field mmf corresponds to field mmf at rated voltage under no load. With the rated voltage along the vertical axis and the rated power factor angle, the phase of rated current is visualized. Then, with  $x_p$  in P.U., the segment AB —  $90^\circ$  ahead of  $I_{1n}$  — the total (airgap) emf at rated load  $E_t$  is found:

$$E_t = \sqrt{v_1^2 + x_p^2 i_1^2 + 2 \cdot x_p \cdot i_1 \cdot v_1 \cdot \sin \phi_n} \text{ (P.U.)} \quad (7.130)$$

Then, the segment CD in the no-load saturation curve represents exactly  $E_t$  and OD (the corresponding field mmf).

Now, we only have to add vectorially to  $\overline{OD}$  the  $\overline{ED}$  in phase with  $I_{1n}$  but with its value (in P.U.).

$F_{an}$  is defined as the ratio between the stator pole rated mmf divided by the field mmf, with  $F_{10n}$  corresponding to rated voltage  $v_n = 1$  (P.U.) at no load.

Rotating OE until it reaches the abscissa give  $OF = OE = F_{1n}$  — the rated field mmf.

It is well understood that the whole method could be put into algebraic form and integrated into a rather simple computer program that calculates first the no-load saturation curve by advancing in 0.05

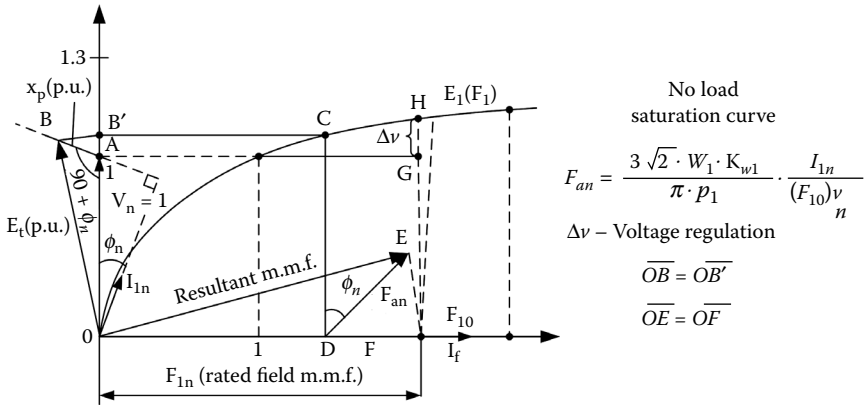


FIGURE 7.24 Rated field magnetomotive force (mmf) calculation.

(or so) steps from zero to 130% of rated airgap flux/pole (or no-load voltage). This way, the graphical errors are removed to a great extent.

The vectorial addition of field (OD) and armature (DE) mmfs per pole to reach the resultant mmf is implicitly valid only if the SG rotor has no magnetic saliency. Even the cylindrical rotor, with slots in axis  $q$  (only), to house the field coils, has an up to 5% saliency;  $x_{dm}/x_{qm} = 1.05$  under full load and rated power factor.

For the salient-pole rotor, the saliency  $x_{dm}/x_{qm} = 1.3 \div 1.5$ , and such an approximation is no longer practical.

This is how the partial magnetization curve method becomes necessary.

### 7.11.2 Partial Magnetization Curve Method

Within the frame of this method, the partial no-load magnetization curves (Figure 7.25) are first determined point by point up to 1.3 P.U. voltage or (flux).

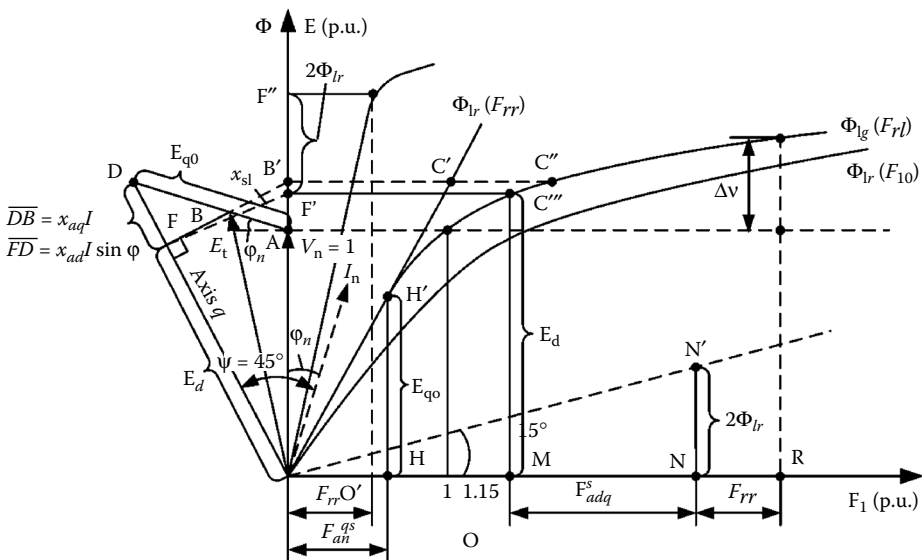
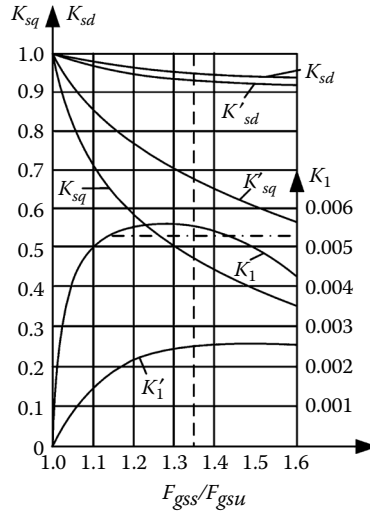


FIGURE 7.25 The partial magnetization curves method.



**FIGURE 7.26** Saturation coefficients to account for cross-coupling magnetic saturation. (Redrawn from V.V. Dombrowski, A.G. Eremeev, N.P. Ivanov, P.M. Ipatov, M.I. Kaplan, and G.B. Pinski, *Design of Hydrogenerators*, vols. I and II, Energy Publishers, Moscow, 1965 [in Russian].)

It is well known that magnetic saturation and saliency produce an angle shifting between the resultant airgap flux and armature and resultant mmf. The cross-coupling magnetic saturation is responsible for this phenomenon (Chapter 4). In Reference [11], a rather lucrative procedure to account for this phenomenon is introduced.

Figure 7.25 shows the phasor diagram similar to Figure 7.24, with  $E_t$  the rated airgap emf  $E_t = \overline{OB'}$ . To  $E_p$ , the unsaturated (straight line)  $\Phi_{1g}$  ( $F_{rl}$ ) retains the unsaturated  $B'C'$  and saturated  $B'C''$ -stator plus airgap mmfs  $F_{rlu}$ ,  $F_{rls}$ :

$$\frac{F_{rlu}}{F_{rls}} = \frac{\overline{B'C'}}{\overline{B'C''}} \quad (7.131)$$

At this ratio, from Figure 7.26 [7], we extract the saturation coefficients  $K_{sd}$ ,  $K_{sq}$ ,  $K_1$  for constant airgap ( $g_{\max} = g$ ) and  $K'_{sd}$ ,  $K'_{sq}$ ,  $K'_1$  for variable airgap ( $g_{\max}/g = 1.5 - 2.5$ ).

Then, for rated armature mmf  $F_{an}$  (P.U.), Figure 7.24, a  $q$  axis equivalent armature reaction occurring for saliency and saturation is calculated:

$$F_{an}^{qs} = F_{an}(\text{P.U.}) \cdot K_{sq} \cdot K_{aq} \quad (7.132)$$

For  $F_{an}^{qs}$ , we read on the  $\Phi_{1g}$  ( $F_{rl}$ ) curve the fictitious emf  $E_{qo} = \overline{HH'}$ .  $E_{qo}$  is projected as BD along the leakage reactance voltage drop direction (AB). The direction OD corresponds to the  $q$  axis. The perpendicular from B to OD corresponds to the  $d$  axis. The perpendicular from B to OD touches the latter in F and OF =  $E_d$  (resultant emf along axis  $d$ ).

To  $E_d$ , on the  $\Phi_{1g}$  ( $F_{rl}$ ) curve, the mmf  $AC'' = OM$  corresponds.

The magnetic saturation corresponding effect is considered by an equivalent  $F_{adq}^s$  component [9]:

$$F_{adq}^s = K_{sd} \cdot K_{ad} \cdot F_a(\text{P.U.}) \cdot \sin \Psi + K_1 \cdot \frac{\tau_p}{g} \cdot F_a(\text{P.U.}) \cdot \cos \Psi \quad (7.133)$$

for constant airgap, and

$$F_{adq}^s = K'_{sd} \cdot K_{ad} \cdot F_a(\text{P.U.}) \cdot \sin \Psi + K'_1 \cdot \frac{\tau_p}{g} \cdot F_a(\text{P.U.}) \cdot \cos \Psi$$

for variable airgap, with  $K_{sd}$ ,  $K'_{sd}$ ,  $K_1$ ,  $K'_1$  from Figure 7.26. The angle  $\Psi$  is the phase angle between the armature current vector and axis  $q$  in the  $d$ - $q$  model. Equation 7.133 contains the influence of both axes along the axis  $d$ .

With  $F_a(\text{P.U.})$  known (for rated point, Figure 7.24) and  $\Psi$  determined from Figure 7.25,  $F_{adq}^s$  is calculated and added to OM along the horizontal axis ( $F_{adq}^s = \text{MM}$ ). Adding the rotor interpole leakage flux per pole ( $2\Phi_{lr} = \text{NN}'$ ), from the  $\Phi_{lr}$  ( $F_{rr}$ ) partial magnetization curve, the rotor mmf contribution  $F_{rr} = \text{OO}'$ , the total on-load excitation mmf is obtained as  $\text{OR} = F_{1n}$ .

Corresponding to  $F_{1n}$ , the voltage regulation  $\Delta v$  (P.U.) is also obtained. The graphical procedure seems at first a bit complicated, but it may be acquired after one to two examples. Also, the procedure may be mechanized into a computer program including the calculation of partial magnetization curves.

It may be argued that the whole problem may be solved directly through FEM. To do so, first,  $E_t$  must be calculated (from Equation 7.122) and then airgap flux  $\Phi_{lg}$  can be calculated with given (rated) stator current and assigned values of  $\Psi$ , and then  $\Phi_{lg}$  can be put into P.U. values. The FEM process can then be gone through again with new values of  $\Psi$  until  $\Phi_{lg}(\text{P.U.}) = E_t$ . The advantage of FEM is the possibility of additionally calculating the slot leakage inductance. In that way, only the end connection leakage inductance is needed in order to obtain an exact value of  $x_{ls}$ .

Still, FEM seems practical only in the design refinement stages rather than in the general optimization design process, due to prohibitively high computation times and a lack of generality of results.

### Example 7.7

Consider the no-load saturation curves in Figure 7.25 as pertaining to a real SG with a leakage reactance  $x_{sl} = 0.11$  and a rated power factor angle  $\phi_n = 20^\circ$ .

Also, the no-load excitation mmf  $F_{10n}$  that produces the rated voltage at zero load is as follows (from Example 7.5):  $F_{10n} = 18242$  At/pole, SCR = 0.7.

Calculate, for a salient pole rotor  $\tau_p/\tau = 0.7$ ,  $g/\tau = 0.04$ , with constant airgap, the rated load excitation mmf in P.U. and in At/pole.

### Solution

First, apply the Potier diagram method, despite the fact that SG has salient poles. From Equation 7.129,

$$x_p \approx x_{sl} + 0.02 = 0.11 + 0.02 = 0.13$$

The airgap emf at full load  $E_p$  for  $V_1 = 1$  (P.U.),  $i_1 = 1$  (P.U.), is, from Equation 7.130:

$$E_t = \sqrt{1 + 0.13^2 \cdot 1^2 + 2 \cdot 0.13 \cdot 1 \cdot 1 \cdot \sin 20} = 1.0515 \text{ (P.U.)}$$

From Figure 7.24, at scale for  $E_t = 1.0515$ , from the no-load saturation curve, the resultant mmf  $\text{OD} = 1.3$  (due to magnetic saturation).

Now, the rated armature  $F_{an}$  is

$$F_{an} = \frac{F_{10n}}{\text{SCR}} = \frac{18242}{0.7} = 26060 \text{ At/pole} = 1.35 \cdot F_{10n}$$

So, in P.U.,  $F_{an} = 1.35$  (P.U.).

Finally, from Figure 7.24, solving for triangle CDF the total load field mmf  $F_{1n} = \overline{OE} = \overline{OF}$  is as follows:

$$F_{1n} = \sqrt{\overline{OD}^2 + \overline{DE}^2 + 2 \cdot \overline{OD} \cdot \overline{DE} \cdot \cos \phi_n} = \sqrt{1.3^2 + 1.428^2 + 2 \cdot 1.3 \cdot 1.428 \cdot \cos 20} = 2.68 \text{ (P.U.)}$$

The value of field rated mmf is, thus,

$$F_{1n} = F_{1n}(\text{P.U.}) \cdot F_{10n} = 2.68 \cdot 18242 = 48876 \text{ At/pole}$$

As expected, no reference was made to the rotor saliency, as the Potier diagram method was used.

Let us now turn to Figure 7.25 and consider that for  $E_p$ , again, the mmf saturation ratio is as follows (Equation 7.131):

$$\frac{F_{qs}}{F_{qsu}} = \frac{\overline{B'C''}}{\overline{B'C'}} = 1.35$$

This saturation ratio corresponds in Figure 7.26, for constant airgap under rotor pole, to

$$K_{sd} = 0.95, K_{sq} = 0.46, K_1 = 0.55 \times 10^{-2}$$

Now, for the rated mmf  $F_{an}$  (1.35 in P.U.), the equivalent armature reaction allowing for saturation and saliency  $F_{an}^{qs}$  is as follows (Equation 7.132):

$$F_{an}^{qs} = F_{an}(\text{P.U.}) \cdot K_{sq} \cdot K_{aq} = 1.35 \times 0.46 \cdot 0.57 = 0.354 \text{ (P.U.)}$$

The values of  $K_{ad}$  and  $K_{aq}$  are given in Figure 7.27 for constant and variable airgap under rotor pole, for given  $\tau_p/\tau$ .

For  $g/\tau = 0.04$ ,  $\tau_p/\tau = 0.7$ ,  $K_{ad} = 0.84$ , and  $K_{aq} = 0.57$  (Figure 7.27).

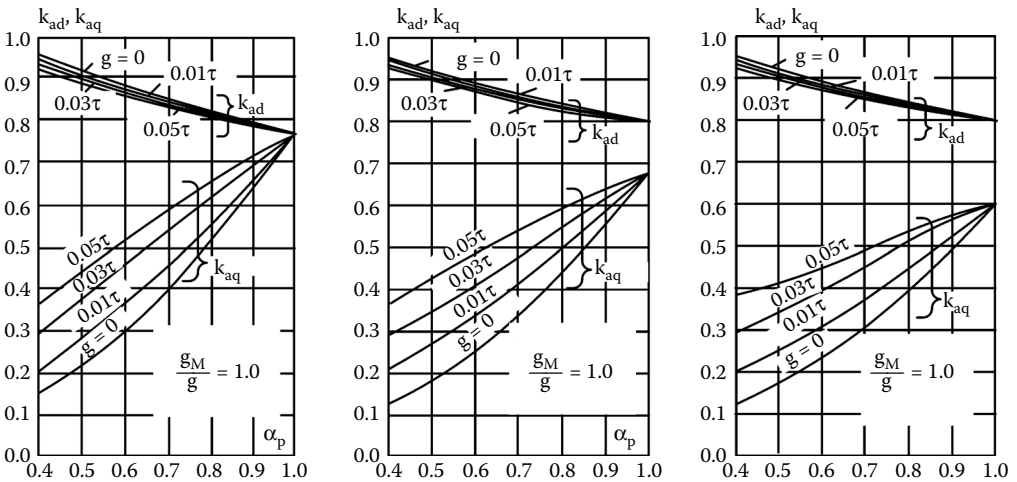


FIGURE 7.27  $K_{ad}$  and  $K_{aq}$  (saliency) reactance factors for various  $\tau_p/\tau$ ,  $g_{\max}/g$ , and  $g/\tau$  values.

For  $F_{an}^{qs} = 0.354$ , we read from the  $\Phi_{1g}$  ( $F_{g0}$ ) the value of emf  $E_{q0} = HH' \approx 0.40 = \overline{BD}$ . From triangle OBD, we determine point F (Figure 7.25); thus,  $E_d = \overline{OF} = \overline{OF'}$ . For  $E_d$ , again, from  $\Phi_{1g}$  ( $F_{rr}$ ), we determine point M, which corresponds to about 1.15 P.U. (Figure 7.25).

With current angle  $\Psi$  to  $q$  axis known, we are now able to calculate from Equation 7.133 the cross-coupling global magnetic saturation effect mmf  $F_{adq}^s$ :

$$\begin{aligned} F_{adq}^s &= K_{sd} \cdot K_{ad} \cdot F_a (\text{P.U.}) \cdot \sin \Psi + K_1 \cdot \frac{\tau_p}{g} \cdot F_a (\text{P.U.}) \cdot \cos \Psi \\ &= 0.95 \cdot 0.84 \cdot 1.35 \cdot \sin 45 + 0.55 \cdot 10^{-2} \cdot \frac{2}{3} \cdot \frac{1}{0.04} \cdot 1.3 \cdot \cos 45 = 0.7616 + 0.0834 = 0.845 \end{aligned}$$

$F_{adq}^s = \overline{MN}$  along the horizontal axis in Figure 7.25, so the rotor leakage flux  $2\Phi_{lr} = \overline{NN'}$  that corresponds to it is about 0.35. This value is equal to  $F'F''$  along the vertical axis ( $F'F'' = 0.35$ ) which, from the  $\Phi_{1r}$  ( $F_{rr}$ ) curves, leads to a rotor mmf  $F_{rr} \approx 0.30$ .

Now,  $F_{rr}$  is added along the horizontal axis as  $\overline{NR}$ .

The load field mmf is, thus,  $F_{ln} = \overline{OR} = \overline{OM} + \overline{MN} + \overline{NR} = 1.15 + 0.845 + 0.30 = 2.295$ .

The obtained value (Equation 2.295) is different from (smaller than) that obtained with the Potier diagram method (Equation 2.68). The three no-load magnetization curves were not calculated point by point, so there is no guarantee of which is better.

However, in terms of precision, it is no doubt that the partial magnetization curves method is better, especially as it accounts for cross-coupling saturation and the magnetic saliency of the rotor.

## 7.12 Inductances and Resistances

The inductances and resistances of an SG refer to the following:

- Synchronous magnetizing inductances (reactances):  $L_{ad}$  ( $X_{ad}$ ),  $L_{aq}$  ( $X_{aq}$ )
- Stator phase leakage inductance (reactance):  $L_{sl}$  ( $X_{sl}$ )
- Homopolar stator inductance:  $L_o$  ( $H_o$ )
- Stator phase resistance:  $R_s$  [ $\Omega$ ]
- Rotor cage leakage inductances (reactances):  $L_{Dl}$  ( $X_{Dl}$ ),  $L_{Ql}$  ( $X_{Ql}$ )
- Rotor cage resistances:  $R_{Dl}$ ,  $R_{Ql}$
- Excitation leakage inductance (reactance):  $L_{fl}$  ( $X_{fl}$ )
- Excitation resistance:  $R_f$

### 7.12.1 The Magnetization Inductances $L_{ad}$ , $L_{aq}$

Simplified formulae for  $L_{ad}$ ,  $L_{aq}$  were derived in Chapter 4:

$$\begin{aligned} L_{ad} &= L_m \cdot \frac{(K_{ad} \cdot K_f)}{1 + K_{sd}^e} & X_{ad} &= \omega_1 \cdot L_{ad} \quad ; \quad x_{ad} = X_{dm} \cdot \frac{I_n}{V_n} \\ L_{aq} &= L_m \cdot \frac{(K_{aq} \cdot K_f)}{1 + K_{sq}^e} & X_{aq} &= \omega_1 \cdot L_{aq} \quad ; \quad x_{aq} = X_{qm} \cdot \frac{I_n}{V_n} \\ L_m &= \frac{6\mu_0}{\pi^2} \cdot \frac{\tau \cdot l_i \cdot (W_1 \cdot K_{W1})^2}{g \cdot K_c} & ; \quad K_f &= \text{from (7.107)} \end{aligned} \quad (7.134)$$



$I_n$ ,  $V_n$  are the base (rated) RMS values of SG phase current and voltage.  $L_m$  represents the airgap inductance for the uniform airgap machine. The airgap  $g$  in  $L_m$  expression is the minimum airgap for variable airgap under rotor poles. The saliency coefficients  $K_{ad}$  and  $K_{aq}$  are given in Figure 7.27. The total magnetic saturation coefficients  $K_{sd}^e$  and  $K_{sq}^e$  are distinct from those in Figure 7.26, as the latter ones are in relation to the stator plus airgap part of mmf.

$K_{sd}^e$  and  $K_{sq}^e$  are both dependent on stator current  $I_1$  and the current angle  $\Psi$  with  $q$  axis; that is, on both  $I_d = I_1 \cdot \sin\Psi$  and  $I_q = I_1 \cdot \cos\Psi$ . Apparently, only FEM is precise enough for calculating  $K_{sd}^e$ ,  $K_{sq}^e$  family curves, as also pointed out in Chapter 5 on transients. When  $I_q = 0$  ( $\Psi = 90^\circ$ ),  $K_{sd}^e(I_f)$  may be determined directly from the no-load saturation curve. But this is only a particular case.

From the no-load curve, the magnetization reactances  $x_{ad}$ ,  $x_{aq}$  in P.U. of the direct and quadrature ( $d$ - $q$ ) axes may be obtained directly from Figure 7.25 as follows:

$$\begin{aligned} x_{aq} \cdot i \cdot \cos\Psi &= \overline{DB} \\ x_{ad} \cdot i \cdot \sin\Psi &= \overline{FD} \end{aligned} \quad (7.135)$$

As Figure 7.25 may be drawn (or translated in algebraic form) for any value of current  $i$  (P.U.) and power factor angle  $\varphi_n$ , the values of  $x_{ad}$  and  $x_{aq}$  for pairs of  $i_d$ ,  $i_q$  ( $i_d = i \cdot \sin\Psi$ ,  $i_q = i \cdot \cos\Psi$ ) may be obtained for given voltage. From the same rationale, the required excitation mmf may be calculated for each situation, as can the power angle  $\delta_v = \Psi - \varphi$ .

### 7.12.2 Stator Leakage Inductance $L_{sl}$

The stator leakage inductance of SG has four components, as it is the case for induction machines [10 ]:

$$L_{sl} = L_{ssl} + L_{szl} + L_{sel} + L_{sdl} \quad ; \quad X_{sl} = \omega_1 \cdot L_{sl} \quad ; \quad x_{sl} = L_{sl} \cdot \omega_1 \cdot \frac{I_n}{V_n} \quad (7.136)$$

where

- $L_{ssl}$  = the slot leakage
- $L_{szl}$  = the (airgap) leakage
- $L_{sel}$  = the end connection leakage
- $L_{sdl}$  = the differential (harmonics) leakage

Consider the two-layer winding with rectangular stator slots, which is typical for SGs (Figure 7.28). In general, shorted coils are used; thus, the currents in the upper and lower layer coils are dephased by  $\gamma_K$  (in general,  $\gamma_K = 60^\circ$  or zero). The angle  $\gamma_K$  is zero in slots where the same phase occupies both layers.

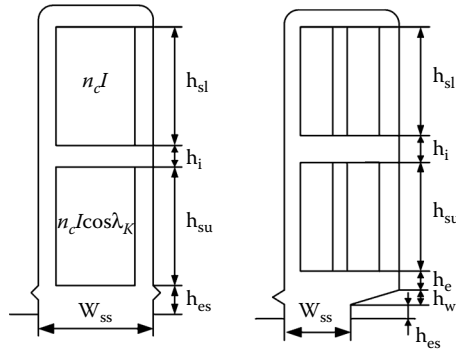


FIGURE 7.28 Typical high- and medium-power synchronous generator stator slots.

As both  $\gamma_K = 0^\circ$  and  $\gamma_K = 60^\circ$  exist, an average of the slot geometrical permeance is to be used.  $L_{sl}$  may be written as follows [10]:

$$L_{sl} \approx 2\mu_0 \cdot \frac{W_1^2 \cdot l}{p_1 \cdot q} (\lambda_{ss} + \lambda_{sz} + \lambda_{se} + \lambda_{sd}) \quad (7.137)$$

with the slot leakage permeance ratio:

$$\lambda_{ss} = \frac{1}{4} \left[ \frac{h_{sl} + h_{su} \cos^2 \gamma_K}{3W_{ss}} + \frac{h_{su}}{W_{ss}} + \frac{h_{su} \cos \gamma_K}{W_{ss}} + \frac{h_i}{W_{ss}} + (1 + \cos \gamma_K)^2 \left( \frac{h_{os}}{W_{ss}} + \frac{h_W}{W_{ss}} + \frac{h_o}{W_{ss}} \right) \right] \quad (7.138)$$

When the number of turns per coil is different in the two layers ( $n_{cl} \neq n_{cu}$ ), Equation 7.138 may be corrected by doing the following:

- Replacing  $\cos \gamma_K$  by  $K \cos \gamma_K$  with  $K = n_{cu}/n_{cl}$
- Replacing number 4 by  $(1 + K)^2$

For the zigzag (z) permeance ratio  $\lambda_{sz}$ , the so-called Richter formula is applied:

$$\lambda_{sz} \approx \frac{5g \cdot K_c / W_{ss}}{5 + 4g \cdot K_c / W_{ss}} \frac{(3\beta_y + 1)}{y} ; \quad \beta_y = \frac{y}{\tau} = \text{coil span ratio} \quad (7.139)$$

It is sometimes inferred that the differential (harmonics) permeance ratio  $\lambda_{sd}$  is included in Equation 7.139. The end-connection permeance ratio  $\lambda_{se}$  (for double-layer winding) is as follows:

$$\lambda_{se} \approx 0.34 \cdot \frac{g}{l_i} \cdot (l_{ec} - 0.64 \cdot y) ; \quad y - \text{coil span} \quad (7.140)$$

with  $l_{ec}$  equal to the end-connection length per machine side. A few remarks are in order:

- The total geometrical (rather than iron) length of stator  $l$  was used, though it includes the radial channels length, to account for the “slot leakage” of coil parts corresponding to cooling channels. It is an overestimation, and better approximations are welcome.
- Alternative formulae for the z permeance ratio were proposed [11]:

$$\lambda_{sz} = \frac{1}{\pi} \cdot l_{er} \cdot \frac{\sqrt{1 + \left( \frac{2g \cdot K_c}{W_{ss}} \right)^2}}{2} + \frac{g \cdot K_c}{W_{ss}} \left( 1 - \frac{2}{\pi} \cdot \tan^{-1} \frac{2 \cdot g \cdot K_c}{W_{ss}} \right) \quad (7.141)$$

Expression 7.141 of  $\lambda_{sz}$  produces large errors for small values of  $g \cdot K_c / W_{ss}$ , which is not the case, in general, for SG.

- There are reasons to consider separately the differential harmonics permeance ratio  $\lambda_{sd}$ . For standard chorded coil winding [10],

$$\sigma_{dso} \approx \frac{2\pi^2}{9 \cdot K_{W1}^2} \cdot \frac{5g^2 + 1 - \frac{3}{4} \left( 1 - \frac{y}{\tau} \right) \left[ 9q^2 \left( 1 - \frac{y^2}{\tau^2} \right) + 1 \right]}{12q^2} \quad (7.142)$$

$$\lambda_{sd} = \sigma_{dso} \cdot \frac{3}{\pi^2} \cdot \frac{\tau \cdot q}{K_C \cdot g} \cdot \frac{l_i}{l} \quad (7.143)$$

- It should be noticed that, as for induction machines (IMs), the solid rotor or the rotor cage may attenuate the differential leakage coefficient  $\lambda_{sd}$  and  $\sigma_{dso}$  “in exchange” for additional losses in the rotor under SG load.
- However, as the slot pitch is about the same in the rotor cage and in the stator, and the airgap tends to be large, this attenuation is limited. Values of attenuation factor of 0.8 to 0.5 might be expected in special cases.
- The end-connection leakage permeance  $\lambda_{se}$  depends much on the exact shape of end connections, their vicinity to iron (metallic) parts of the frame and to the stator stack end plate, and even on the value of field mmf (the power factor).
- Saturation of stator end core laminations that produces additional losses in the underexcited machine, also causes a reduction in the end-connection leakage permeance ratio  $\lambda_{se}$ , due to induced current effects. And so do the currents induced in the neighboring metallic parts.
- Three-dimensional FEM was used to compute the end-connection leakage flux and losses in the end core and in the metallic parts nearby. The results seem satisfactory for the case in point but lack generality.
- The case of variable airgap above the rotor pole shoe further complicates the computation of both zigzag and differential leakage inductance. As usual, FEM is the solution for refined calculations, but in the preliminary design stages, the above formulae give results with  $\pm 10\%$  error, in general, and are fairly reliable.

## 7.13 Excitation Winding Inductances

As already pointed out in [Chapter 4](#), when reduction to stator is performed, the mutual inductance (reactance) between the field-winding and stator phases,  $L_{fa}(x_{fa})$ , equals the  $d$  axis magnetization inductance,  $L_{ad}(x_{ad})$ . However, in the design and in the testing process, it is useful to first calculate the main and leakage excitation inductance, before the reduction to stator.

The main field inductance  $L_{fg}$  and the leakage field inductance  $L_{fl}$  make up the excitation total inductance, as seen (or measured) from the rotor:

$$L_f = L_{fg} + L_{fl} \quad (7.144)$$

$L_{fg}$  is simply as follows (as for  $L_{ad}$ ):

$$L_{fg} = \frac{2p_1 \cdot \mu_0 \cdot W_f^2 \cdot \tau \cdot l_i \cdot \frac{\tau_p}{\tau}}{g \cdot K_C \cdot (1 + K_{sd})} \quad (7.145)$$

The stator reduction coefficient  $K_{fa}$  is

$$K_{fa} = \frac{3}{2} \cdot \left( \frac{W_1 \cdot K_{w1}}{2p_1 \cdot W_f} \right)^2 \cdot \left( \frac{4}{\pi} \cdot K_{ad} \right)^2 \quad (7.146)$$

On the other hand, the actual mutual inductance between the excitation and armature windings,  $M_{af}$ , is

$$M_{af} \approx 6 \cdot W_f \cdot W_1 \cdot K_{W1} \cdot K_f \cdot \frac{2\mu_0 \cdot \tau \cdot l_i}{\pi \cdot g \cdot K_C \cdot (1 + K_{sd})} \quad (7.147)$$

The leakage excitation inductance  $L_{fl}$  also contains a few terms:

$$L_{fl} = 2 p_1 \cdot \mu_0 \cdot W_f^2 \cdot l_p \cdot (\lambda_p + \lambda_{zf} + \lambda_{ef}) \quad (7.148)$$

$L_{fl}$  components have similar formulae as those derived for the stator.

The slot leakage permeance ratio  $\lambda_p$  was calculated in Equation 7.138;  $\lambda_{zf}$  was calculated in Equation 7.139.

The end-connection permeance ratio  $\lambda_{ef}$  is as follows [11]:

$$\lambda_{ef} \approx 2.55 \cdot \frac{a_{ps}}{l_i} \cdot \ln \left( 1 + \frac{\pi}{4} \cdot \frac{\tau_p}{2b_{r1}} \right) \quad (7.149)$$

with  $b_{r1}$ ,  $a_{ps}$  from Figure 7.22.

The reduction of  $L_{fl}$  to the stator makes use of  $K_{fa}$  of Equation 7.146, while the P.U. translation is done by dividing to  $L_B = (V_n/I_n \cdot \omega_1)$ :

$$l_{fl} = L_{fl} \cdot K_{fa} / L_B = \frac{L_{ad} \cdot \pi \cdot g \cdot q \cdot K_C}{3 \cdot \mu_0 \cdot K_{W1}^2 \cdot K_{ad} \cdot l_i \cdot \tau} \cdot (\lambda_p + \lambda_{zf} + \lambda_{ef}) \quad (7.150)$$

Now it is easier to add the differential component missing in Equation 7.148. The differential leakage is similar to the case of stator winding:

$$l_{fl} \rightarrow l_{fl} + l_{ad} \cdot \left( 2 \frac{\tau_p}{\tau} \cdot \frac{K_{ad}}{K_{f1}^2} - 1 \right) \quad (7.151)$$

$K_{f1}$  was calculated in Equation 7.107 for constant airgap under the pole shoes. It is common practice to express the base inductance (reactance)  $L_B$  ( $X_B$ ) based on the following approximations:

$$V_n \approx \pi \sqrt{2} \cdot f_1 \cdot W_1 \cdot K_{W1} \cdot \Phi_{10} \quad (7.152)$$

with

$$\Phi_{10} = \frac{2}{\pi} \cdot B_{g10} \cdot \tau \cdot l_i$$

$B_{g10}$  corresponds to no-load conditions at rated voltage.

$$F_{a1} = \frac{3\sqrt{2} \cdot W_1 \cdot K_{W1} \cdot I_n}{\pi \cdot p_1} \quad (7.153)$$

as

$$X_b = \frac{V_n}{I_n} = \omega_1 \cdot l_b = \frac{3}{\pi} \cdot \frac{W_1}{p_1} \cdot \frac{(W_1 K_{w1})^2}{p_1} \cdot \frac{\Phi_{10}}{F_{an1}} \quad (7.154)$$

By denoting  $F_{gn}$  as the airgap mmf requirement,

$$F_{gn1} = \frac{B_{g1}}{K_{f1}} \cdot \frac{g \cdot K_C}{\mu_0} \quad (7.155)$$

The magnetization reactance  $x_{ad}$  and  $x_{aq}$  in (P.U.) become

$$x_{ad} = \frac{K_{ad}}{K_{f1}} \cdot \frac{F_{an1}}{F_{gn1}} \quad (7.156)$$

$$x_{aq} = \frac{K_{aq}}{K_{f1}} \cdot \frac{F_{an1}}{F_{gn1}} \quad (7.157)$$

with  $x_{ad}$  given in Equation 7.156, together with the airgap flux density  $B_{g1}$  and linear current loading  $A_n = 3W \cdot I_n / (p_1 \cdot \omega)$ , the airgap  $g$  may be found:

$$g = \frac{2 \cdot \mu_0 \cdot K_{ad} \cdot K_{w1}}{x_{ad} \cdot K_{f1} \cdot K_C} \cdot \frac{A_n \cdot \tau}{B_{gn1}} \quad (7.158)$$

This expression was used to size the airgap earlier in this chapter (Equation 7.67).

Note that for the cylindrical rotor, the leakage inductance formula is similar to Equation 7.148, but instead of  $2p_1 \cdot W_f^2$ , it will be  $2p_1 \cdot N_{fp} \cdot W_{fc}^2$ , where  $W_{fc}$  is the number of turns per rotor slot, and  $N_{fp}$  is the number of field-winding slots per pole in the rotor.

While the slot and zigzag permeances are straightforward, the end-connection permeance ratio  $\lambda_{ef}$  is as follows:

$$\lambda_{ef} \approx \frac{0.2 \times \tau_r}{l_i} \quad (7.159)$$

$\tau_r$  = the rotor excitation slot pitch.

## 7.14 Damper Winding Parameters

The  $d$  and  $q$  axes damper winding leakage reactances  $x_{D\sigma}$  and  $x_{Q\sigma}$  in P.U. have expressions similar to those of excitation ( $l_{\beta}$ ) [12,13]:

$$x_{Dl} \approx x_{ad} \cdot \left[ \frac{K_{ad}}{\pi \cdot K_D^2} \cdot \frac{\alpha_b \cdot N_2 \cdot (1 - K_b)}{4 \sin^2 \frac{\alpha_b}{2}} \cdot \left( 2 + \frac{\cos N_2 \alpha_b - K_b \cos \alpha_b}{1 - K_b} \right) \cdot \left( 1 + \frac{2 \cdot \lambda_{De} \cdot g \cdot K_C}{l_i} \right) - 1 + \frac{N_2 \cdot (1 - K_b) \cdot K_{ad} \cdot l_{pi} \cdot g \cdot K_C}{l_i \cdot \tau \cdot K_D^2} \cdot (\lambda_{sD} + \lambda_{zD}) \right] \quad (7.160)$$

$$\alpha_b = \frac{\pi \cdot \tau_b}{\tau} \quad ; \quad \tau_b - \text{damper bar pitch}$$

$$K_b = \frac{\sin(N_2 \alpha_b)}{N_2 \sin \alpha_b}$$

$$K_D \approx \frac{N_2}{\pi} \cdot (1 - K_b)$$
(7.161)

with  $\lambda_{De}$  equal to the end-ring permeance ratio. Complete end-ring presence is supposed:

$$\lambda_{De} \approx \frac{3.32}{\pi} \cdot \log \frac{2.35 \cdot D_{ring}}{(a + 2b)}$$
(7.162)

where

$D_{ring}$  = the average ring diameter

$a, b$  = the cross-section ring dimensions:  $a$  is radial and  $b$  is axial

The zigzag and damper slot permeance ratios  $\lambda_{ZD}$  and  $\lambda_{SD}$  are straightforward.

For axis  $q$ ,

$$x_{ql} = x_{ql} \cdot \left\{ \frac{K_{aq}}{\pi K_Q^2} \frac{\alpha_b N_2}{4 \sin^2 \frac{\alpha_b}{2}} \cdot \left[ \cos N_2 \alpha_b - K_b \cos \alpha_b + \frac{\alpha_i \pi + \pi(1 - \alpha_i) \frac{g K_c}{2 g_{\max}}}{N_2 \cdot \alpha_b} \right] \times \right.$$

$$\left. \times \left( 1 + 2 \cdot \lambda_{De} \cdot \frac{g \cdot K_c}{l_i} \right) \right\} - 1 + \frac{N_2 \cdot (1 + K_b) \cdot K_{aq} \cdot l_{pi} \cdot g \cdot K_c}{l_i \cdot \tau \cdot K_Q^2} (\lambda_{sQ} + \lambda_{zQ}) \left\{ \right.$$
(7.163)

with

$$K_Q \approx \frac{N_2}{\pi} \cdot (1 + K_b) - \frac{4}{\pi} \cdot \left( 1 - \frac{4}{\pi} \frac{\sigma \cdot g \cdot K_c}{g_{\max}} \right) \cdot \cos \frac{\alpha_i \cdot \pi}{2} \cdot \frac{\sin \left( \frac{N_2}{2} \cdot \alpha_b \right)}{2 \sin(\alpha_b/2)}$$
(7.164)

where  $\alpha_i = \tau_p / \tau$  is the rotor pole span per pole pitch.

The damper cage resistance in P.U.,  $r_D$  and  $r_Q$ , are as follows:

$$r_D \approx \frac{6(W_1 \cdot K_{W1} \cdot K_{ad})^2 \cdot \rho_D \cdot N_2 \cdot (1 - K_b)}{\pi^2 \cdot p_1 \cdot K_D^2 \cdot X_b} \cdot \left[ \frac{l_{pi}}{A_{bar}} + \frac{\alpha_b \cdot \tau}{2\pi \cdot A_{ring} \cdot \sin^2 \frac{\alpha_b}{2}} \times \right.$$

$$\left. \times \left( 2 + \frac{\cos N_2 \alpha_b - K_b \cos \alpha_b}{1 - K_b} \right) \right]$$
(7.165)

$$r_Q \approx \frac{6(W_1 \cdot K_{W1} \cdot K_{aq})^2 \cdot \rho_D \cdot N_2}{\pi^2 \cdot p_1 \cdot K_Q^2 \cdot X_b} \cdot \left\{ \frac{l_{pi}}{A_{bar}} (1 + K_b) + \frac{\alpha_b \cdot \tau}{2\pi \cdot A_{ring} \cdot \sin^2 \frac{\alpha_b}{2}} \times \right. \\ \left. \times \left[ \cos N_2 \alpha_b - K_b \cos \alpha_b + \frac{\pi}{N_2 \cdot \alpha_b} \cdot (1 - \cos N_2 \alpha_b) \right] \right\} \quad (7.166)$$

Note that for incomplete end-rings, in general,

$$x_{Ql} = (2 + 3) x_{Dl} \quad ; \quad R_{Ql} = (4 + 6) R_{Dl} \quad (7.167)$$

where

$\rho_D$  = the damper cage resistivity ( $\Omega m$ )  
 $A_{bar}$  = the damper bar cross-section ( $m^2$ )  
 $A_{ring}$  = the end-ring cross-section ( $m^2$ )

The field-winding and stator resistances in P.U. are as follows:

$$r_f = \frac{R_f}{X_b} \cdot \frac{6(W_1 \cdot K_{W1} \cdot K_{ad})^2}{(\pi \cdot p_1 \cdot W_f \cdot K_{f1})^2} \quad ; \quad R_f = \rho_{cor} \cdot l_f \cdot \frac{W_f}{A_{cof}} \cdot 2 p_1 \quad (7.168)$$

where

$R_f$  = the actual field-winding resistance ( $\Omega$ )  
 $W_f = N_{fp} \cdot W_{fc}$  for the cylindrical rotor  
 $l_f$  = the average field turn length  
 $A_{cof}$  = the field turn cross-section

$$r_s = \frac{R_s}{X_b} \quad ; \quad R_s = \rho_{cos} \cdot W_a \cdot \frac{l_{coil} \cdot W_a}{A_{cos} \cdot a} \cdot K_R \quad (7.169)$$

where

$l_{coil}$  = the turn length  
 $A_{cos}$  = the stator turn cross-section  
 $K_R$  = the skin effect coefficient (to be determined in the paragraph on losses)  
 $a$  = the stator current paths number  
 $W_a$  = the turns/path

## 7.15 Solid Rotor Parameters

Cylindrical rotors are built of solid iron, and for some salient-pole SGs, solid rotor poles may also be used. The solid iron of the rotor acts as a damper cage with variable resistance and inductance (reactance).

The solid iron parameters depend on the frequency of induced rotor currents —  $Sf_1$  for the fundamental and  $2f_1$  for the inverse components.

The presence of field-winding slots in the solid iron poles makes the  $d$  and  $q$  axes equivalent parameters of solid iron dampers different from each other and difficult to calculate. Comprehensive analytical formulae with widespread acceptance for  $r_D$ ,  $r_Q$ ,  $l_{Dp}$ ,  $l_{Ql}$  for solid iron poles are not yet available, but

$$x_{Dl} \approx 0.6 \cdot r_{Dl} \quad ; \quad x_{Ql} \approx 0.6 \cdot r_{Ql} \quad (7.170)$$

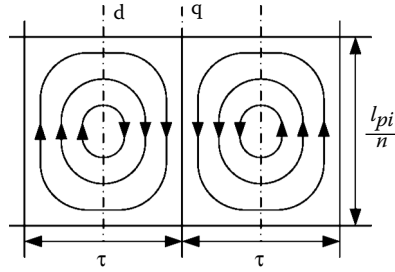


FIGURE 7.29 Solid iron rotor eddy currents trajectory in axis  $d$ .

The trajectory of eddy currents induced by the mmf space harmonics (Figure 7.29) and stator slot opening airgap conductance harmonics or during transients depends on the pole pitch of the respective harmonics, the frequency as “sensed” by the rotor currents, and level of flux density in the rotor solid iron body.

For the mmf fundamental only, the frequency of rotor-induced currents is  $Sf_1$  [ $S = (\omega_1 - \omega_r)/\omega_1$ ].

The depth of field penetration for a traveling wave with pole pitch  $\tau$  is as follows [10]:

$$\delta_1 = \text{Real} \sqrt{\frac{1}{\left(\frac{\pi}{\tau}\right)^2 + j\mu_{Fe}\sigma_{Fe} \cdot 2\pi \frac{Sf_1}{K_t}}} \quad (7.171)$$

$K_t$  takes into account the conductivity reduction due to the eddy current tangential closure [10] — the so-called transverse cage effect:

$$K_t \approx \frac{1}{1 - \left[ \tan\left(\frac{\pi}{\tau} \cdot \frac{l_{pi}}{2 \cdot n}\right) \right] / \frac{\pi}{\tau} \cdot \frac{l_{pi}}{2 \cdot n}} \quad (7.172)$$

$n$  is the number of solid iron rings put together axially to form the rotor.

The iron permeability  $\mu_{Fe}$  is “dictated” by the “steady-state” conditions. So, the procedures used to calculate the magnetic saturation curves and rated excitation mmf also yield at least an average value for  $\mu_{Fe}$ . To a first approximation, on the rotor surface, the flux density is, in general, around 1 to 1.2 T in the unslotted region and 1.5 to 1.8 T in the slotted region of cylindrical rotors. Magnetic saturation leads to a decrease in  $\mu_{Fe}$  and, thus, to a larger field penetration depth. That is, to a smaller equivalent resistance.

For an unslotted rotor, the solid iron resistance reduced to the stator is as follows [10]:

$$R_{Ds} = \frac{6(W_1 \cdot K_{W1})^2 \cdot l_{pi} \cdot K_t}{\sigma_{Fe} \cdot p_1 \cdot \tau \cdot \delta_1} \quad ; \quad r_D = \frac{R_{Ds}}{X_B} \quad (7.173)$$

We may, to a first approximation, consider  $R_{Ds} = R_{Qs}$ , and allow for the airgap leakage:

$$x_{Ds} \approx x_{Qs} \approx 0.6 \cdot r_{Ds} + 0.035 \quad (7.174)$$

## 7.16 SG Transient Parameters and Time Constants

The  $d$  axis transient reactance  $x'_d$  is as follows:



$$x'_d = x_{sl} + \frac{1}{\frac{1}{x_{ad}} + \frac{1}{x_{fl}}} \text{ (P.U.)} \quad (7.175)$$

The transient reactance of the  $d$  axis damper cage,  $x'_D$ , is as follows:

$$x'_D = x_{Dl} + \frac{1}{\frac{1}{x_{ad}} + \frac{1}{x_{fl}}} \text{ (P.U.)} \quad (7.176)$$

The subtransient  $d$ - $q$  axis reactance  $x''_d$  is

$$x''_d = x_{sl} + \frac{1}{\frac{1}{x_{ad}} + \frac{1}{x_{Dl}} + \frac{1}{x_{fl}}} \text{ (P.U.)} \quad (7.177)$$

For the  $q$  axis,  $x''_q$  is

$$x''_q = x_{sl} + \frac{1}{\frac{1}{x_{aq}} + \frac{1}{x_{ql}}} \text{ (P.U.)} \quad (7.178)$$

The total excitation time constant  $T_f$  is

$$T_f \approx \frac{x_{ad} + x_{fl}}{r_f \cdot \omega_n} \text{ (seconds)} \quad (7.179)$$

The transient short-circuit  $d$  axis time constant  $T'_d$  is

$$T'_d = T_f \cdot \frac{x'_d}{x_d} \text{ (seconds)} ; x_d = x_{sl} + x_{ad} \quad (7.180)$$

The  $d$  axis damping time constant  $T_D$ , with open stator and short-circuited superconducting ( $r_f = 0$ ) field circuit, is

$$T_D = \frac{x'_D}{r_D \cdot \omega_n} \text{ (seconds)} \quad (7.181)$$

The subtransient  $d$  axis time constant  $T''_d$  is as follows:

$$T''_d = T_D \cdot \frac{x''_d}{x'_d} = \frac{x'_D}{r_D \cdot \omega_n} \cdot \frac{x''_d}{x'_d} \text{ (seconds)} \quad (7.182)$$

If the  $q$  axis damper winding is considered in isolation, its time constant  $T_Q$  is

$$T_Q \approx \frac{x_{Ql} + x_{aq}}{r_Q \cdot \omega_n} \text{ (seconds)} \quad (7.183)$$

The subtransient  $q$  axis time constant (the stator and the field windings are short-circuited and superconducting)  $T_q''$  is

$$T_q'' = T_Q \cdot \frac{x_q''}{x_q} = \frac{(x_{Ql} + x_{aq})}{(x_{sl} + x_{aq})} \cdot \frac{x_q''}{r_Q \cdot \omega_n} \text{ (in seconds)} \quad (7.184)$$

The negative sequence reactance  $x_2$  is

$$x_2 = \frac{x_d'' + x_q''}{2} \quad (7.185)$$

or

$$x_2 = \sqrt{x_d'' \cdot x_q''} \quad (7.186)$$

depending on whether negative sequence voltages or currents are present in the SG stator.

The time constant  $T_a$  of the stator currents when all other windings are superconducting and short-circuited is as follows:

$$T_a = \frac{2 x_d'' \cdot x_q''}{r_s (x_d'' + x_q'') \cdot \omega_n} \quad (7.187)$$

All the above parameters appear in the sudden short-circuit time response. As the sudden three short-circuit is used to measure (estimate) the above parameters, their expressions, as derived in the previous paragraph, serve for checking the design accuracy and predicting the SG transient behavior.

### 7.16.1 Homopolar Reactance and Resistance

The homopolar sequence ([Chapter 4](#)) does not produce interference with the rotor in terms of the fundamental. However, it produces a fixed (AC) magnetic field in the airgap with a pole pitch  $\tau_3 = \tau/3$ , similar to a third-space harmonic. So, ACs are induced in the rotor cage through this AC third-harmonic field. In general, this effect is neglected, and the homopolar reactance  $x_o$  is assimilated to a stator leakage inductance calculated with slot total currents either zero or twice the coil homopolar mmf:  $2n_c \cdot I_o$ . This is so, as homopolar currents in all phases are the same. To a first approximation,

$$\begin{aligned} x_o &\approx x_{sl} \text{ for diametrical coils} \\ x_o &\approx 3 \cdot \left(1 - \frac{\gamma}{\tau}\right) \cdot x_{sl} \text{ for chorded coils} \end{aligned} \quad (7.188)$$

More complicated expressions are found in the literature [6].

It is evident that Equation 7.188 ignores the damping effect of rotor damper-induced currents produced by the homopolar stator mmf. In such conditions, the homopolar resistance  $r_o \approx r_s$ .

**Example 7.8: The Transient Reactances and Time Constants**

A designed salient-pole SG has the following calculated parameters (in P.U.):  $r_s = 0.003$ ,  $r_f = 0.006$ ,  $x_{ad} = 1.5$ ,  $x_{aq} = 0.9$ ,  $x_{sl} = 0.12$ ,  $x_{fl} = 0.15$ ,  $x_{Dl} = 0.04$ ,  $x_{Ql} = 0.05$ ,  $r_D = 0.02$ , and  $r_Q = 0.022$ .

Calculate the transient  $x'_d$  and subtransient reactances  $x''_d$ ,  $x''_q$ , then  $\times 2$  in P.U. and the time constants  $T'_d$ ,  $T''_d$ ,  $T''_q$ ,  $T_a$ ,  $T_D$  in seconds for  $\omega_r = 2\pi \cdot 60$  rad/sec.

*Solution*

From Equation 7.175, the subtransient  $d$  axis reactance  $x'_d$  is as follows:

$$x'_d = x_{sl} + \frac{1}{\frac{1}{x_{ad}} + \frac{1}{x_{fl}}} = 0.12 + \frac{1}{\frac{1}{1.5} + \frac{1}{0.15}} = 0.25636 \text{ P.U.}$$

From Equation 7.177,  $x''_d$  is as follows:

$$x''_d = x_{sl} + \frac{1}{\frac{1}{x_{ad}} + \frac{1}{x_{Dl}} + \frac{1}{x_{fl}}} = 0.12 + \frac{1}{\frac{1}{1.5} + \frac{1}{0.04} + \frac{1}{0.15}} = 0.1509 \text{ P.U.}$$

From Equation 7.178,

$$x''_q = x_{sl} + \frac{1}{\frac{1}{x_{aq}} + \frac{1}{x_{Ql}}} = 0.12 + \frac{1}{\frac{1}{0.9} + \frac{1}{0.05}} = 0.16736 \text{ P.U.}$$

$$x_2 = \frac{x''_d + x''_q}{2} = \frac{0.1509 + 0.16736}{2} = 0.15913 \text{ P.U.}$$

or

$$x_2 = \sqrt{x''_d \cdot x''_q} = \sqrt{0.1509 \cdot 0.16736} = 0.158917 \text{ P.U.}$$

The time constants are calculated from Equation 7.180 through Equation 7.184 and Equation 7.187:

$$T'_d = T_f \cdot \frac{x'_d}{x_d} = \frac{1.5 \cdot 0.15}{0.006 \cdot 2\pi \cdot 60} \cdot \frac{0.25636}{(1.5 + 0.12)} = 0.115 \text{ (seconds)}$$

$$T_D = \frac{\left( x_{Dl} + \frac{1}{1/x_{ad} + 1/x_{fl}} \right)}{r_D \cdot \omega_n} = \frac{\left( 0.05 + \frac{1}{1/1.5 + 1/0.15} \right)}{0.02 \cdot 2\pi \cdot 60} = 0.024716 \text{ (seconds)}$$

$$T''_d = T_D \cdot \frac{x''_d}{x'_d} = 0.024716 \cdot \frac{0.1509}{0.25636} = 0.01458 \text{ (seconds)}$$

$$T_q'' = \frac{(x_{ql} + x_{aq})}{r_Q \cdot \omega_n} \cdot \frac{x_q''}{x_q} = \frac{(0.05 + 0.9)}{0.022 \cdot 2\pi \cdot 60} \cdot \frac{0.1676}{0.9} = 0.02133 \text{ (seconds)}$$

$$T_a = \frac{2x_d'' \cdot x_q''}{r_s(x_d'' + x_q'') \cdot \omega_n} = \frac{0.1509 \cdot 0.16736}{0.003 \times 0.15913 \cdot 2\pi \cdot 60} = 0.140 \text{ (seconds)}$$

The total no-load excitation time constant  $T_f$  (Equation 7.179) is as follows:

$$T_f = \frac{x_{ad} + x_{fl}}{r_f \cdot \omega_n} = \frac{1.5 + 0.15}{0.006 \cdot 2\pi \cdot 60} = 0.7254 \text{ (seconds)}$$

## 7.17 Electromagnetic Field Time Harmonics

In order to perform well, when connected to the power system, the open-circuit voltage waveform has to be very close to a sine wave. Standards limit the time harmonics, traditionally through the so-called telephonic harmonic factor (THF): 5% up to 1 MW, 3% up to 5 MW, and 1.5% above 5 MW per unit.

Today, “grid codes” specify total harmonic distortion (THD) to 1.5% for SGs in near 400 kV power systems and 2% in near 275 kV power systems. There are proposals to raise these limits to 3% (3.5%).

To analyze possibilities to reduce emf THD, let us start with its expression:

$$E = \omega_1 \cdot \sum_{v=1}^{\infty} K_{W_v} \cdot W_a \cdot \Phi_v$$

$$\Phi_v = \frac{2}{\pi} \cdot \frac{\tau}{v} \cdot B_{gv} \cdot l_i \quad (7.189)$$

$$K_{W_v} = K_{dv} K_{yv} = \frac{\sin(v\pi/6)}{q \cdot \sin(v\pi/6q)} \cdot \sin\left(\frac{v \cdot y}{\tau} \cdot \frac{\pi}{2}\right)$$

For fractionary  $q$  windings,

$$q = b + c/d = (bd + c)/d$$

$$K_{dv} = \frac{\sin(v\pi/6)}{(bd + c) \cdot \sin(v\pi/6(bd + c))} \quad (7.190)$$

So, the harmonics occur in the airgap flux density ( $B_{g1}$ ) first.

They may be reduced by the following:

- Adjusting the ratio of rotor pole shoe span  $\tau_p$  per pole pitch  $\tau$ :  $\alpha_i = \tau_p/\tau$  in salient pole rotors
- Varying the airgap from  $g$  to  $g_{\max}$  from center to margins in the salient pole rotor shoe
- Adjusting the large (central) tooth  $\tau_p$  per pole pitch  $\tau$  ratio in nonsalient pole rotors with uniform airgap

The form (harmonics) coefficients in the excitation airgap flux density for constant airgap are as follows:

$$K_{fv} \approx \frac{4}{\pi \cdot v} \sin \left( v \cdot \frac{\tau_p}{\tau} \cdot \frac{2}{\pi} \right) - \text{salient poles} \quad (7.191)$$

$$K_{fv} \approx \frac{4}{\pi^2 \cdot v^2} \frac{\cos \left( v \cdot \frac{\tau_p}{\tau} \cdot \frac{2}{\pi} \right)}{1 - v \cdot \frac{\tau_p}{\tau} \cdot \frac{2}{\pi}} - \text{nonsalient poles} \quad (7.192)$$

It seems that the minimum of harmonics content

$$\frac{\sum_{v>1} E_v^2}{E_1} = \frac{\sum K_{fv}^2}{K_{f1}} \quad (7.193)$$

caused solely by the airgap flux density harmonics, is obtained in the first case (salient poles) for  $\alpha_p = \frac{\tau_p}{\tau} = 0.77 - 0.8$ . However, these large values produce too large an interpole excitation flux leakage, and  $\alpha_i \approx 0.7$  is practiced, although occasionally larger.

For the nonsalient poles,  $\tau_p/\tau \approx 1/3$  seems adequate, but the precise ratio  $\tau_p/\tau$  may be used to destroy a certain harmonics  $v$ :

$$v \cdot \frac{\tau_p}{\tau} \cdot \frac{2}{\pi} = (2K+1) \cdot \frac{\pi}{2} \quad (7.194)$$

As even harmonics do not normally occur (all pole geometry, airgap, and excitation coils are identical), the harmonics of interest are  $v = 3, 5, 7, 9, 11, 13, 15, 17, 19, \dots$ . The first slot-opening-caused harmonic pair  $v_c = 6q \pm 1$  may also be the target, especially with integer  $q < 5$ .

For salient pole machines, the airgap may be varied ideally as follows:

$$\delta(\theta) = \frac{\delta}{\cos p_1 \theta} \quad (7.195)$$

For such a case,

$$(K_{fv})_{g \sin} \approx \frac{2}{\pi} \cdot \left[ \frac{\sin(v-1)\alpha_p \cdot \pi/2}{v-1} + \frac{\sin(v+1)\alpha_p \cdot \pi/2}{v+1} \right] \quad (7.196)$$

A notable reduction in the fifth and seventh harmonics is obtained (Table 7.2). The augmentation of the third harmonic is not a problem in a three-phase machine.

**TABLE 7.2** Airgap Flux Density Harmonics ( $\alpha_p = 2/3$ )

|                  | $K_{f1}$ | $K_{f3}$ | $K_{f5}$ | $K_{f7}$ |
|------------------|----------|----------|----------|----------|
| Constant gap     | 1.105    | 0        | 0.221    | +0.158   |
| Inverse sine gap | 0.941    | 0.137    | 0.137    | +0.069   |

In reality, airgap variation under pole shoes is caused by lower radius rotor pole machining. FEM should be used to quantify the “exact” effect on airgap flux harmonics.

The next step in reducing emf harmonics is the proper design of the stator winding. One solution is the reduction of  $K_{dv}$  and  $K_{\nu}$  for key harmonics.

An increase in integer  $q$  above five does not produce a notable decrease in  $(K_{dv})_{v \geq 5}$ , not to mention  $K_{d3}$ , which is almost independent of  $q$  and large (around 0.67). But,  $q = 2, 3$  produces significantly higher harmonics. The worst situation,  $K_{dv_c} = 1$ , takes place for the slot harmonics  $v_c = 6K_q \pm 1$ .

Raising the order of the first slot harmonics implies larger integer  $q$  ( $q \geq 5$ ) or fractionary  $q = b + c/d = (bd + c)/d$ . In this case, the first slot harmonic is  $v_c = 6(bd + c) \pm 1$ .

One more way to reduce the emf harmonics is to cancel (reduce) the fifth and seventh harmonics by chorded coils two-layer windings:

$$\sin\left(v \cdot \frac{y}{\tau} \cdot \frac{\pi}{2}\right) = 0 \quad (7.197)$$

$$v \cdot \frac{y}{\tau} \cdot \frac{\pi}{2} = K_n \cdot \pi \quad (7.198)$$

For  $y/\tau = 0.833 = 5/6$ , the best reduction of fifth and seventh harmonics takes place:

$$K_{y5} = \sin\left(5 \cdot 0.833 \cdot \frac{\pi}{2}\right) = 0.256$$

$$K_{y7} = \sin\left(7 \cdot 0.833 \cdot \frac{\pi}{2}\right) = 0.2624$$

A coil chording of about 5/6 may be obtained easily with a large  $q$ .

The third harmonic has to be kept below 5% of the fundamental to avoid false tripping of relays for differential protection when an SG is connected or disconnected to a bus bar, with both systems earthed to allow for triple-frequency currents to flow to the ground through neutral points.

It should be mentioned that the stator mmf harmonics due to the location of stator coils in slots produces mmf harmonics of orders  $v = 6K \pm 1$ , that is,  $-5, +7, -11, +13$ . These space harmonics do not produce time harmonics in the emf, but produce additional losses in the rotor. This is why they have to be reduced through increased or fractionary  $q$ .

## 7.18 Slot Ripple Time Harmonics

In certain conditions, the airgap permeance variation due to slot openings may produce higher harmonics, known as slot ripple, in the open-circuit emf wave.

The stator slot opening airgap magnetic permeance in interaction with the field mmf fundamental produces the slot ripple emf effect (Figure 7.30):

The excitation (rotor) mmf fundamental  $F_{f1}$ , with respect to stator, is written as follows:

$$F_{f1}(\theta_s) = F_{f1m} \cdot \sin(p_1 \cdot \theta_s - \omega_r \cdot t) \quad (7.199)$$

The airgap magnetic conductance variation is

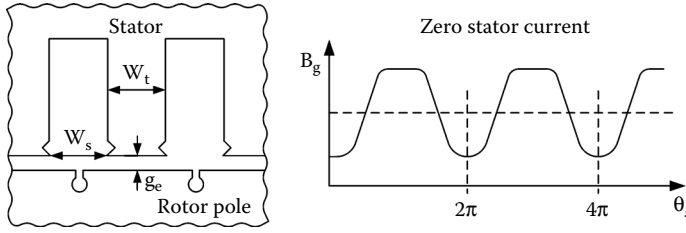


FIGURE 7.30 Slot-opening permeance variation-caused flux density pulsations.

$$P_g = \frac{\mu_0 \cdot K_{fm}}{g} \cdot \sin v_1 N_s \theta_s \quad (7.200)$$

The airgap flux density  $B_g$  is

$$\begin{aligned} B_g &= P_g \cdot F_{f1} = \\ &= \frac{\mu_0 \cdot K_{fm}}{g} \cdot F_{f1m} \cdot \left\{ \cos \left[ (v_1 N_s - p_1) \theta_s + p_1 \cdot \omega \cdot t \right] - \cos \left[ (v_1 N_s + p_1) \theta_s - p_1 \cdot \omega \cdot t \right] \right\} \end{aligned} \quad (7.201)$$

For  $v = 1$ , the first airgap permeance harmonic is considered, while for  $v_1 = 2$ , the second one comes into play. Both are important.

If the rotor damper bar pitch  $\tau_d$  corresponds to a division of circle by  $N_s - p_1$  or  $N_s + p_1$ , then the currents induced by the flux density pulsation due to  $v_1 = 1$  in the rotor bars are zero. This is why  $\tau_s = \tau_r$  (stator slot pitch  $\approx$  damper bar pitch) produces good results.

In addition, with a complete end-ring, rotor-damper-bar-induced currents may flow between adjacent poles if

$$\frac{N_s}{p_1} \pm 1 = 2K_o \pm 1 \quad (7.202)$$

That is, when  $N_s/p_1$  is an even integer number, there will be induced currents in the rotor bars, due to stator slotting, between poles. These rotor-bar-induced interpolar currents produce their own mmfs:

$$F_{1d} = K_{1d} \cdot \left[ \cos(p_1 \theta_s - v_1 N_s \omega t) - \cos(p_1 \theta_s + v_1 N_s \omega t) \right] \quad (7.203)$$

This mmf produces its own reaction flux density in the airgap:

$$B_{d1} = K_{1d} \cdot \left[ \cos(p_1 \theta_s - (v_1 N_s + p_1) \omega t) - \cos(p_1 \theta_s - (v_1 N_s - p_1) \omega t) \right] \quad (7.204)$$

It is now clear that this density produces emf time harmonics of the frequency  $f'$ :

$$f' = \frac{\omega}{2\pi} \cdot \left( \frac{v_1 N_s}{p_1} \pm 1 \right) \quad (7.205)$$

To avoid that such time emf harmonics would, in turn, induce pole-to-pole currents in the rotor cage, we need to fulfill the following condition:

$$\frac{v_1 N_s}{p_1} \pm 1 = 2K_o \pm \frac{1}{2} \quad (7.206)$$

Such a condition may be met by displacing the groups of bars on each pole from the center line such that the displacement of cage bars between adjacent poles should be  $1/2$  stator slot pitch.

Now, if the  $N_s/p_1$  ratio is an odd number, the condition in Equation 7.206 is fulfilled automatically. This is not so for the second slot harmonic ( $v_1 = 2$ ) when  $2N_s/p_1$  is equal to an even number.

With  $W_s/g_e > 2$  and  $W_r/W_s > 1.5$ , the second permeance harmonic may be large. To avoid the slot ripple mmf for  $v_1 = 2$ , this simply means that in choosing the rotor bar pitch  $\tau_d$  should be half the stator slot pitch  $\tau_s$ . As this would seem to be too many rotor bars, skewing the stator slots by one half might be the solution.

Fractionary  $q$  tends to eliminate slot ripple influence on the mmf, with  $\tau_s \approx \tau_r$ .

Note that while the above rationale produces intuitive suggestions to reduce emf time harmonics, their verification is now possible using FEM.

## 7.19 Losses and Efficiency

Four main categories of losses occur in SGs:

- Winding (copper)
- Core (iron)
- Mechanical
- Brush-ring electrical

The winding (copper) losses include stator winding losses and excitation losses (plus exciter losses if the exciter is mounted on the SG shaft).

Core (iron) losses include the stator core fundamental (50[60] Hz) hysteresis, eddy current losses, and additional core losses.

Additional core losses are produced by the field winding mmf (under no load, also):

- On the rotor surface
- On the stator end laminations stacks, in the tightening plate, and, by the stator mmf space harmonics, again on the rotor core surface

Another way to classify the electromagnetic losses is to consider them as no-load losses and short-circuit losses (at given current). Such a division of losses corresponds to practical tests on SGs, where such losses may be measured.

Despite the fact that there are notable differences between short-circuit and full load in terms of magnetic saturation, the combination of no-load and short-circuit losses proves to be adequate in industry. There are approximate analytical expressions to calculate the various components of no-load and short-circuit losses that have stood the test of time in industry. In parallel, FEM was recently applied to calculate various components of losses in SGs.

At least for rotor surface losses, it was recently proved that analytical methods [14] correlate very well with FEM results [15].

### 7.19.1 No-Load Core Losses of Excited SGs

The no-load core losses of SG consist of the following:

- Fundamental stator core hysteresis losses,  $p_h$
- Eddy current losses,  $p_{edo}$
- Additional rotor pole shoe losses,  $p_{pso}$
- Additional stator core losses in the lamination stacks  $p_{eFe10}$



There are also additional “core” losses in the stator tightening end plates or pressure fingers and in the axial stator bolts that tighten the stator stack, but these are relatively smaller in a proper design.

The fundamental stator core losses  $P_{Fe10}$  are as follows:

$$P_{Fe10} = p_h + p_{edy} = \left[ K_h \cdot p_{h0} \cdot \left( \frac{f}{50} \right) + K_{ed} \cdot p_{edo} \cdot \left( \frac{f}{50} \right)^2 \right] \cdot \left( \frac{B}{1.0} \right)^2 \cdot G \quad (7.207)$$

where

$p_{h0}$  and  $p_{edo}$  = the specific hysteresis and eddy current losses in 1 kg of lamination at 1.0 T and 50 Hz, respectively

$G$  = the weight of laminations material

As known, the flux density  $B$  is far from uniform in the stator teeth or yoke. Still, an average has to be considered in Equation 7.207. Even Equation 7.207 is a bit impractical, as it needs separation of hysteresis and eddy current losses, which is not directly available in single frequency tests.

So, in general,

$$P_{Fe10} \approx p_{10} \cdot \left( \frac{f}{50} \right)^{a_f} \cdot \left( \frac{B}{1.0} \right)^{a_b} \cdot G \quad (7.208)$$

where the coefficients  $a_f$  and  $a_b$  may be found from curve-fitting the losses for a wide range of  $f$  (frequency) and  $B$  values.

Considering a uniform flux density distribution in stator teeth and yoke, the total stator core losses are as follows:

$$(P_{Fe10})_t = p_{10} \cdot \left( \frac{f}{50} \right)^{1.3} \cdot \left[ K_{y1} \left( \frac{B_{y1}}{1.0} \right)^2 G_{y1} + K_{t1} \left( \frac{B_{t1/30}}{1.0} \right)^2 G_{t1} \right] \quad (7.209)$$

$B_{t1/30}$  is the tooth flux density at 30% slot height.

$K_{y1}$  and  $K_{t1}$  are empirical factors allowing for loss augmentation due to machining the laminations ( $K_{y1} \approx 1.3 - 1.6$ ,  $K_{t1} \approx 1.8 - 2.4$ ).  $G_{y1}$ ,  $G_{t1}$  are the stator yoke and teeth weights. Finally,  $p_{10}$  is the specific loss (in watts per kilogram [W/kg]) at 1.0 T and 50 Hz.

When FEM flux distribution is calculated, it is possible to consider the instantaneous values of flux density in each volume element and to add up the losses in these elements. Still, as the machining factors are so important, FEM results in core losses have not yet produced spectacular results in terms of precision over analytical expressions that were corrected over time, based on industrial experience.

Though it is known that for the same frequency and flux density, the specific iron losses (in W/kg) differ in AC and traveling fields, and  $p_{10}$  is still obtained from AC field measurements. At least in the stator yoke, the field is mainly of traveling character.

One more question remains here: is it worth it and how do we consider the traveling field and AC field loss regions when determining the stator core losses?

The rotor surface no-load losses  $p_{ps0}$  are produced by the field-current-caused airgap flux density variation on the rotor pole surface due to stator slot openings.

The frequency of currents induced on the rotor surface is  $f_{ps} = f_s \cdot n$ , where  $n$  is the rotor speed. To reduce these losses, the pole shoes are made, whenever possible, from 1 to 1.8 mm thick laminations.

However, in turbogenerators and in high-speed hydrogenerators, the rotors are made of solid iron. In this case, large airgap  $g$  per stator slot opening  $W_{ss}$  ratios ( $g/W_{ss} \geq 1$ ) are adopted to secure low flux density pulsations due to slot opening, and thus, moderate pole surface losses result.

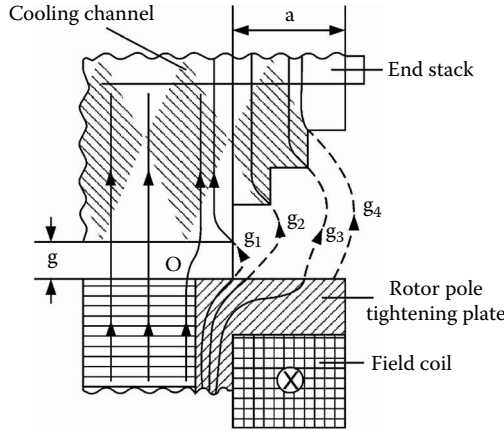


FIGURE 7.31 Excitation flux lines in the end stack of the stator.

Though the situation differs to some extent in salient pole, in contrast to cylindrical pole, rotors, general analytical expressions were produced:

$$p_{pso} = 0.232 \cdot 10^6 \Delta \left[ (K_{C_1} - 1) \cdot B_{g0} \cdot \tau_s \right]^2 \cdot 2 p_1 \cdot A_{pole} \cdot (N_s \cdot n)^{1.5} \quad (7.210)$$

where

- $\Delta$  = the lamination thickness in the pole shoe [m]
- $K_{C_1}$  = the stator slotting Carter coefficient
- $B_{g0}$  = the no-load airgap flux density [T]
- $\tau_s$  = the stator slot pitch [m]
- $A_{pole}$  = the rotor pole shoe area [m<sup>2</sup>]
- $N_s$  = the number of stator slots
- $n$  = the rotor speed in rps

This expression is hardly valid for solid rotor poles, when the depth of field penetration is not only dependent on the frequency ( $N_s \cdot n$ ) but also on flux density level (due to magnetic saturation).

As the stator mmf space harmonics also produce eddy currents on the surfaces of rotor solid poles, we will treat this case in the paragraph on short-circuit (load) losses.

### 7.19.2 No-Load Losses in the Stator Core End Stacks

As seen in Figure 7.31, a part of airgap excitation flux paths closes through the rotor pole tightening plates. If the latter are long enough, the flux lines meet the end-stack laminations in the stator at 90°. Consequently, the lamination effect does not work anymore, and considerable iron losses are produced in the end stacks. Stepping the airgap in the end stack reduces these losses to some extent.

Considering that the stator and the rotor surfaces are magnetically equipotential, the flux densities  $B_{g1}$ ,  $B_{g2}$ ,  $B_{g3}$ , corresponding to the airgaps  $g_1$ ,  $g_2$ ,  $g_3$  are as follows:

$$B_{g1} = B_{g0} \cdot \frac{g \cdot K_c}{g_1}$$

$$B_{g2} = B_{g0} \cdot \frac{g \cdot K_c}{g_2} \quad (7.211)$$

$$B_{g3} = B_{g0} \cdot \frac{g \cdot K_c}{g_3}$$

The airgaps  $g_1$ ,  $g_2$ ,  $g_3$  are considered half-circles; thus, their lengths are straightforward if the width of end-stack step length  $a/n$  is known:

An average axial flux density  $B_m$  is

$$B_m = \sqrt{\frac{B_{g1}^2 + 4B_{g2}^2 + B_{g3}^2}{6}} \quad (7.212)$$

The approximate (classical) formula for the end-stack losses in the teeth zone is

$$p_{ezo} \approx \frac{142 \times 10^{-3}}{\rho_{lam}} \cdot (B_m \cdot W_{ta})^2 \cdot G_{t1} \quad (7.213)$$

$W_{ta}$  = the width of stator teeth — an average of it.

The teeth may be split in the middle radially, and then  $W_{ta} = W_{t1}/2$ ; thus, the end-stack core losses are notably reduced. The lamination resistivity  $\rho_{lam} \approx 0.5 \times 10^{-6} \Omega m$  and increases with temperature.

The total no-load core losses  $p_{Feo}$  are as follows:

$$p_{Feo} = p_{Fe10} + p_{pso} + p_{ezo} \quad (7.214)$$

### 7.19.3 Short-Circuit Losses

The short-circuit losses contain quite a few components. The first one is the stator winding loss.

The stator winding (upper) AC resistance loss is as follows:

$$p_{co_{sc}} = 3(R_s)_{dc} \cdot K_R \cdot I_1^2 \quad (7.215)$$

The coefficient  $K_R > 1$  accounts for the frequency (skin) effect on stator resistance.

By design, generally,  $K_R < 1.33$ , even in large power SGs. In large power SGs, to keep  $K_R$  less than 1.33, Roebel bar single-turn windings are used. In Roebel bars, the single turn, divided into many elementary conductors in parallel, uses its transposition along the stack length such that all of the divisions occupy all positions within the turn location; thus, the circulation currents between elementary conductors is zero.

With the circulating currents made zero through transposition, the skin effect coefficient  $K_R$  is the same as that for the elementary conductors in series (same current in all):

$$K_{Ra} = \varphi(\xi) + \frac{(n_l^2 - 1)}{3} \cdot \Psi(\xi) \quad (7.216)$$

$$\varphi(\xi) = \xi \cdot \frac{\sinh 2\xi + \sin 2\xi}{\cosh 2\xi - \cos 2\xi}; \quad \Psi(\xi) = 2\xi \cdot \frac{\sinh 2 - \sin 2\xi}{\cosh 2\xi - \cos 2\xi} \quad (7.217)$$

$$\xi = \alpha \cdot h_1; \quad \alpha = \sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma_{co} \cdot \frac{b_l}{W_s}}$$

where

- $b_1$  = the elementary conductor width per layer in slot
- $n_l$  = the number of layers in a turn
- $h_1$  = the elementary conductor height
- $\tau_{co}$  = the copper electrical conductivity

For two-layer winding with chorded coils:

$$K_{Ra} = \varphi(\xi) + \frac{(n_l^2(5 + 3\cos\beta) - 8)}{24} \cdot \Psi(\xi) \quad (7.218)$$

$\beta = \frac{\gamma}{\tau} \cdot \frac{\pi}{2}$  is the coil span angle.

With some slots hosting the same phases and some different phases, a kind of average value for  $K_{Ra}$  has to be calculated. Notice that for the slots hosting the same phase in both winding layers, the number of elementary conductor layers is  $2n_l$  in Equation 7.216.

The skin effect tends to be much smaller in the end connection, and even without transposition in this zone, the skin effect coefficient is smaller than that in the stack (slot) zone.

To secure  $K_{Ra} \leq 1.33$ , the radial size  $h_1$  of the elementary conductor in the Roebel bar has to be limited.

With the usual number of elementary conductor layers  $n_l$  going up to 12 and even more,  $\xi$  has to be smaller than unity  $\xi < 1$ .

For this case,

$$K_{Ra} \approx 1 + \frac{n_l^2}{9} \cdot \xi^4 \cdot \varphi \quad \text{for single-layer winding} \quad (7.219)$$

and, respectively,

$$K_{Ra} = 1 + n_l^2 \cdot \left( \frac{5}{72} + \frac{1}{24} \cos\beta \right) \cdot \xi^4 \quad \text{for double-layer winding} \quad (7.220)$$

where

$$\begin{aligned} \sigma_{co} &= 4.8 \cdot 10^7 \text{ (}\Omega\text{m)}^{-1} \\ f &= 60 \text{ Hz} \\ b_1/W_s &= 0.64 \end{aligned}$$

$$\alpha = \sqrt{\pi \cdot f \cdot \mu_0 \cdot \sigma_{co} \cdot \frac{b_1}{W_s}} = \sqrt{\pi \cdot 60 \cdot 4.8 \cdot 10^7 \cdot 1.256 \cdot 10^{-6} \cdot 0.64} = 85.26 \text{ (m)}^{-1} \quad (7.221)$$

To limit  $K_{Ra}$  to 1.33, it may be demonstrated that the maximum elementary conductor height  $h_1$  is as follows [10]:

$$h_1 \approx \frac{15 \text{ mm}}{\sqrt{n_l}} \quad (7.222)$$

The total Roebel bar copper height is  $n_l \cdot h_1$ :

$$n_l \cdot h_l = h_{bar} \approx 15 \text{ mm} \times \sqrt{n_l} \quad (7.223)$$

For example, for the number of elementary conductors,  $n_l = 16$  per bar,  $h_{bar} = 15 \text{ mm} \times \sqrt{16} = 60 \text{ mm}$ , and  $h_l = 15/4 = 3.5 \text{ mm}$ .

With two layers in the windings, the total copper height is  $2h_{bar} = 120 \text{ mm}$ . Considering all insulation layers, the total slot height could reach up to 200 mm. As slot width  $W_s \leq (30 \text{ to } 40) \text{ mm}$ , it means that the usual  $h_{slot}/W_s < 6.5$ .

Most practical cases can be handled by Roebel bars up to highest power per unit (1500 MVA at 28 kV line voltage).

In reality, the number of elementary conductor layers may be increased to 48 when their radial heights go down to almost 2 mm, so even deeper slots may be allowed, once conditions (Equation 7.222 and Equation 7.223) are met.

Note that there are many other formulae corresponding to cases different from Roebel bar (characterized by the zero circulating currents). For example, in the literature [6] for the skin effect losses per coil,

$$K'_{Ra} = \frac{n_l^2 \cdot \xi^2}{9} \cdot n_c^2 \quad \text{for bottom coil side} \quad (7.224)$$

$$K'_{Ra} = \frac{7 \cdot n_l^2 \cdot \xi^2}{9} \cdot n_c^2 \quad \text{for top coil side} \quad (7.225)$$

where  $n_c$  is equal to turns/coil ( $n_c = 1$  for single turn/bar coils).

Besides this, there is an extra loss coefficient due to circulating current  $K'_{Rc}$ :

$$K'_{Rc} = \frac{C_e^2 \cdot \xi^4 \cdot n_l^2 \cdot b_e^2}{180} \cdot (1 + 15n_c^2) \cdot K_{trans} \quad (7.226)$$

with  $C_e$  equal to insulated strand height/bare conductor strand height. Also,

$$b_e = \frac{l - \frac{1}{2} n_c \cdot l_c}{l_{ec}} \quad (7.227)$$

where

$l$  = the total stator stack length

$1/2 \cdot n_c \cdot l_c$  = half the cooling channel length

$l_{ec}$  = connection coil length per machine side

There is no transposition in multiturn coils ( $n_c > 1$ ), and then  $K_{trans} = 1$ .

For two-turn coils (semi-Roebel transposition),  $K_{trans} \approx 0.0784$ .

As expected for full Roebel bar transposition,  $n_c = 1$  and  $K_{trans} = 0$ , as discussed earlier in this paragraph.

The total skin effect coefficient  $K_{Ra}$  is as follows:

$$K_{Ra} = 1 + K'_{Ra} + K'_{Rc} \quad (7.228)$$

#### 7.19.4 Third Flux Harmonic Stator Teeth Losses

The third harmonic in airgap flux density, caused jointly by the stator mmf, magnetic saturation, and the field-winding mmf, produces extra iron losses in the stator teeth.

An approximate expression of these losses  $p_{Fe3}$  is as follows:

$$p_{Fe3} \approx 10.7 \times p_{10} \left( \frac{f}{50} \right)^2 \cdot \left( \frac{B_{3\sim}}{1} \right)^{1.25} \cdot G_{t1} \quad (7.229)$$

where

$p_{10}$  = the core losses (W/kg) at 1.0 T and 50 Hz

$G_{t1}$  = the stator teeth weight

$B_{3\sim}$  = the third harmonic of airgap flux

To a first approximation, only

$$B_{3\sim} \approx \frac{(B_{t1})_{av}}{A_{1s}} \cdot (A_{3s} x_d + 1.3 A_{3r} \cdot x_{ad}) \quad (7.230)$$

where

$(B_{t1})_{av}$  = the average flux density in the stator teeth

$A_{3s}$  = the third harmonic of stator mmf per pole (A/m)

$A_{3r}$  = the third harmonic of field mmf per pole (A/m)

$A_{1s}$  = the fundamental of stator mmf per pole (A/m)

Still, the magnetic saturation produced third harmonic, in phase with the fundamental, is not included in Equation 7.230. Also, the third flux density harmonic may be damped by the currents induced in the rotor poles or in damper windings. This attention of  $B_{3\sim}$  is not considered either.

The slot opening airgap permeance harmonics in the airgap flux density, now produced by the stator mmf, cause rotor surface (and cage) losses  $p_{ps}$  (at rated current):

$$p_{ps} \approx K' \cdot \left( \frac{1}{(K_{C1} - 1)} \cdot \frac{2 p_1}{N_s} \cdot x_{ad} \right)^2 \cdot p_{pso} \quad (7.231)$$

where  $p_{pso}$  are the same losses but are produced by the excitation mmf (at no load):  $K' \approx 0.31$ ; for  $g_{\max}/g = 1$ ; 0.2 for  $g_{\max}/g = 1.5$ ; 0.15 for  $g_{\max}/g = 2.0$ ; 0.12 for  $g_{\max}/g = 2.5$ . The fifth, seventh, eleventh, thirteenth, and so on, stator mmf space harmonics produce the losses:

$$p_{ps}^a \approx \frac{2.1}{\sqrt[3]{q}} \cdot \left( \frac{x_{ad}}{(K_{C1} - 1)} \cdot K_{ch} \right)^2 \cdot p_{pso} \quad (7.232)$$

where

$q$  = the slots per pole per phase

$K_{C1}$  = the Carter coefficient due to stator slotting

$K_{ch}$  (depending on coil chording) = shown in [Figure 7.32](#)

In Reference [15], a more exact analytical approach, now verified by FEM and by some experiments, is given for the no-load and on-load solid rotor surface losses: ( $p_{pso}$ ,  $p_{ps}$ ,  $p_{ps}^a$ ).

We will deal with this approach in some detail here, as it seems to be notable progress in the art.

### 7.19.5 No-Load and On-Load Solid Rotor Surface Losses

A two-dimensional multilayer field theory was used to develop an analytical formula for solid rotor pole losses in SGs [14]. The losses per unit area of rotor solid pole are as follows:

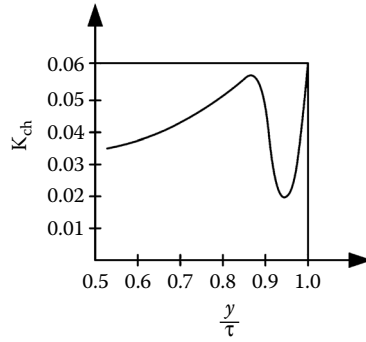


FIGURE 7.32 Coefficient  $K_{ch}$  in Equation 7.232.

$$\frac{p_{ps}}{A_{ps}} = \frac{B_{g1p}^2 \cdot \tau_s^2 \cdot f_{el}^2 \cdot \sigma_{Fe}}{4 \cdot \text{Re}(\gamma_p)} \cdot K_L \quad (7.233)$$

where  $B_{g1p}$  is the first harmonic oscillation flux density in the rotor solid pole. It is a function of airgap no load such as flux density pulsation,  $B_{g10p}$ :

$$B_{g1p} = B_{g10p} \cdot \frac{\left[ (1 - \mu_{r3}) e^{-\frac{2\pi g}{\tau_s}} (\mu_{r1} - 1) + (1 + \mu_{r3}) e^{\frac{2\pi g}{\tau_s}} (\mu_{r1} + 1) \right]}{(1 - \mu_{r3}) e^{-\frac{2\pi g}{\tau_s}} \left( \mu_{r1} - \frac{\tau_s |\gamma_p|}{2\pi} \right) + (1 + \mu_{r3}) e^{\frac{2\pi g}{\tau_s}} \left( \mu_{r1} + \frac{\tau_s |\gamma_p|}{2\pi} \right)} \quad (7.234)$$

$$B_{g10p} = \alpha_1 \cdot B_m$$

with

$$\gamma_p^2 = \left( \frac{2\pi}{\tau_s} \right)^2 - j \cdot \sigma_{Fe} \cdot 2\pi \cdot f_{el} \cdot \mu_0 \cdot \mu_{r,ab} \quad (7.235)$$

The relative permeabilities  $\mu_{r1}$ ,  $\mu_{r3}$  refer to rotor pole, and, respectively, to stator core for  $B_{g10p}$ . In fact,  $B_{g1p}$  replaces  $B_{g10p}$  in Equation 7.234 to account for rotor-pole-induced current damping effect.

The ratio  $K_L$  is the so-called harmonic factor:

$$K_L = \left( \frac{B_m}{B_{g10p}} \right)^2 \sum_{v=1}^{\infty} \frac{1}{\sqrt{v}} \cdot \left( \frac{B_{gvop}}{B_m} \right)^2 \quad (7.236)$$

where

$\alpha_1$  = the flux density oscillation factor (Figure 7.33)

$B_m$  = the mean flux density in the airgap over one stator slot pitch

$f_{el}$  = the electrical frequency of induced currents

$B_{gvop}$  = the peak value of the  $v$ th harmonic flux density at the rotor pole surface

Typical values for  $K_L$  are shown in Figure 7.34 [15].

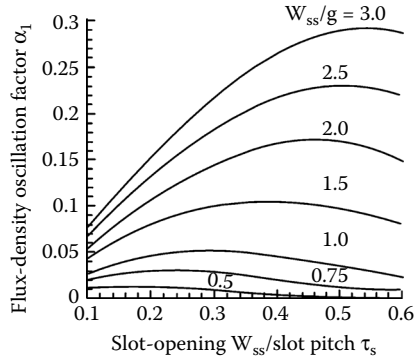


FIGURE 7.33  $\alpha_1 = B_{g10p}/B_m$  ratio: opening/slot.

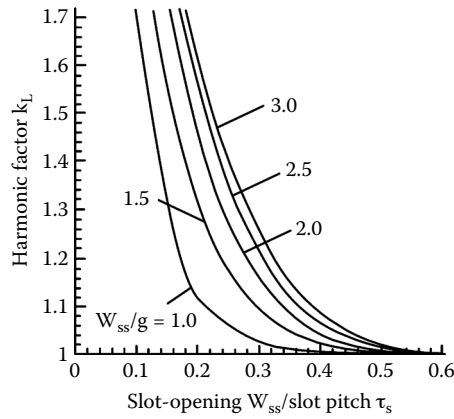


FIGURE 7.34 Harmonic factor  $K_L$  vs. slot opening/airgap ( $W_{ss}/g$ ) and slot pitch  $W_s/\tau_s$ .

Still, in the inverse of depth of penetration formula ( $\gamma_p$  in 7.235), there is the unique relative permeability of iron  $\mu_{r,ab}$  to be determined iteratively from

$$H_o \cdot \sqrt{\mu_0 \cdot \mu_{r,ab}} = \frac{\tau_s}{2\pi} \cdot (0.85 \cdot B_{g1p}) \cdot \left| \gamma_p \right| \cdot \frac{1}{\sqrt{\mu_0 \cdot \mu_{r,ab}}} \quad (7.237)$$

So, the value of  $\mu_{r,ab}(K)$  is calculated for  $0.85 B_{g1p}$  for a given value of  $B_{g10p}$ . With this value, from the magnetic material curve  $B_n/H_n$ , introduced in the right side of Equation 7.237, a new value  $\mu_{r,ab}(K+1)$  is obtained. After a few iterations, with some under relaxation,

$$\begin{aligned} \mu_{r,ab}(K+1) &= \mu_{r,ab}(K) + K_{un} [\mu_{r,ab}(K+1) - \mu_{r,ab}(K)] \\ K_{un} &= 0.2 \div 0.3 \end{aligned} \quad (7.238)$$

Sufficient convergence is obtained.

It was also shown that the ripple loss per unit area of solid rotor is proportional to the following:

$$\frac{P_{ps}}{A_{ps}} \approx B_{g1p}^2 \cdot \tau_s^2 \cdot f_{el}^{1.5} \cdot K_L \cdot \sqrt{\sigma_{Fe}} \quad (7.239)$$



As the tangential field in the rotor pole dominates, the saturation effect is represented by

$$0.85 \cdot B_{g1p} \cdot \frac{\tau_s}{2\pi} \approx \mu_0 \cdot \mu_{r,ab} \cdot H_o \cdot \frac{1}{\sqrt{\pi \cdot f_{el} \cdot \sigma_{Fe} \cdot \mu_0 \cdot \mu_{r,ab}}} \quad (7.240)$$

$$H_o \cdot \sqrt{\mu_0 \cdot \mu_{r,ab}} \approx \sqrt{f_{el} \cdot \sigma_{Fe}} \cdot \tau_s \cdot B_{g1p} = S_D$$

In general, the ripple loss per unit area of solid rotor increases linearly with  $S_D$  [15].

It is recommended here to check, whenever possible, the flux density values in the airgap and in the solid rotor via FEM eddy current software, in order to secure, by fudge factors, a good estimation level of rotor surface losses.

For special (rated) operation modes, FEM may be used directly.

Additional losses occur in the stator tightening plates made of solid metal and in the teeth pressure fingers. There are again traditional analytical expressions for these losses [6], but, after FEM investigations, it seems that their use should be made with extreme care. The total short-circuit losses  $p_{sc}$  are as follows:

$$p_{sc} \approx p_{cos} + p_{Fe3} + p_{pss} \quad (7.241)$$

#### 7.19.5.1 Excitation Losses

Basically, the excitation losses contain the following:

- Field winding DC losses:

$$p_{ex} = R_f \cdot I_f^2 \quad (7.242)$$

- Brush and slip-ring losses (if any):

$$p_b = 2 \cdot \Delta V \cdot I_f ; \Delta V < 1V \text{ by design} \quad (7.243)$$

If a machine exciter is used, the total excitation power as seen from the SG,  $p_{exsh}$ , is as follows:

$$p_{exsh} = \frac{R_f \cdot I_f^2 + 2 \cdot \Delta V \cdot I_f}{\eta_{ex}} \quad (7.244)$$

$\eta_{ex}$  is the total efficiency of the exciter system.

In case of static exciter  $\eta_{ex}$  is the static exciter efficiency. For a brushless exciter, the brush and slip-ring losses are replaced by the rectifier losses, which are, in general, of the same order of magnitudes.

#### 7.19.5.2 Mechanical Losses

Mechanical losses may be approximated to losses at no load, with the SG unexcited ( $I_1 = I_f = 0$ ):

- Bearing losses (axial and guidance bearings in vertical axial SGs)
- Ventilation losses (in the ventilator, its circuit and shaft air friction)

When pumps are used, in direct hydrogen or water cooling, their input power has to be considered.

$$p_{mec} = p_{bear} + p_{vent} + p_{air} \quad (7.245)$$

Simplified formulae for losses were introduced over the years based on experience.

For air-cooled hydrogenerators with vertical shaft, here are a few such expressions:

- The bearing losses  $p_{bear}$  (include both the axial and guidance bearing)

$$p_{bear} = p_{bear}^{axial} + p_{bear}^{guide} \approx \left[ 9.81 \cdot G_V \cdot U_{bear} + F_m + F_d \cdot U_{bear}^a \right] \cdot \mu_f \quad (7.246)$$

where

$G_V$  = the total vertical mass of turbine and generator rotors plus the axial water-push mass in kilograms

$U_{bear}$  = radial bearing peripheral speed

$U_{bear}^a$  = the axial bearing peripheral speed

$F_m$  = the magnetic pull due to estimated rotor eccentricity

$F_d$  = the mechanical radial force due to eccentricity and mechanical rotational unbalance

$$F_m \approx 0.02 \cdot G_{MT+SG} \cdot 9.81$$

$$F_d \approx 1.256 \times 10^6 \cdot B_{gn}^2 \cdot \alpha_i \cdot D \cdot l_i \cdot \frac{e}{g} \quad (7.247)$$

where

$B_{MT+SG}$  = the turbine plus SG rotor mass

$\alpha_i$  = the ideal rotor pole span ratio

$D$  = the stator bore diameter

$l$  = the stator stack iron length

$e$  = the eccentricity

$g$  = the airgap

$B_{gn}$  = the rated airgap flux density

$e/g \approx 0.1$  to  $0.2$

The friction coefficient  $\mu_f$  depends essentially on hydrogenerator speed (in rpm) and on coil temperature at 50°C,  $\mu_f = 0.0035$  at  $n = 50$  rpm, and  $\mu_f = 0.0108$  at  $n = 500$  rpm.

- The ventilator plus circuit losses,  $p_{vent}$ , may be written as follows:

$$p_{vent} \approx 1.1 \cdot Q_{air} \cdot U_{vent}^2 \quad (7.248)$$

$Q_{air}$  is the airflow rate in cubic meters per second required to evacuate the losses from the SG.

$U_{vent}$  is the peripheral average speed of the ventilator.

The rotor air-windage losses  $p_{air}$  is as follows:

$$p_{air} \approx C_a \cdot (2\pi n)^3 \cdot D_r^5 \cdot \left( 1 + \frac{5 \cdot l_p}{D_r} \right) \quad (7.249)$$

where

$C_a \approx (1.5 - 3) \cdot 10^{-3}$  = a machine constant depending on rotor smoothness

$n$  = the rotor speed in rps

$D_r$  = the rotor external diameter

$l_p$  = the total length of rotor poles

The total mechanical loss  $p_{mec}$  is as follows:

$$p_{mec} = p_{bear} + p_{vent} + p_{air} \quad (7.250)$$

The losses in the machine contain the no-load losses at rated voltage and short-circuit losses at rated current plus the mechanical losses and the excitation losses:

$$\sum losses = p_{Feo} + p_{sc} + p_{mec} + p_{exch} \quad (7.251)$$

The load losses may be calculated based on no-load and short-circuit losses by adopting an appropriate value for field current  $I_f$ , stator current  $I_1$  and the power factor angle  $\varphi$ .

On the other hand, the airflow rate  $Q_{air}$  may be calculated from the following:

$$Q_{air} = \frac{\sum losses}{C_a \cdot \Delta T_{air}} \quad (7.252)$$

where

$\sum losses$  = the total SG losses that have to be evacuated by the cooling air

$C_a \approx 1.1 \times 10^3 \text{ Ws/m}^2\text{C}$  = the heat capacity factor of air

$\Delta T_{air}$  = the air warming temperature differential

### 7.19.5.3 SG Efficiency

The SG efficiency is defined as the output (electrical)/input (mechanical) power:

$$\eta_{SG} = \frac{(P_2)_{electric}}{(P_1)_{mechanical}} = \frac{(P_2)_{electric}}{(P_2)_{electric} + \sum losses} \quad (7.253)$$

In industry, the efficiency is calculated for five load ratios: 25%, 50%, 75%, 100%, and 125%.

The voltage and speed are constant, so the no-load losses (or iron losses, mainly)  $p_{Feo}$  and mechanical losses  $p_{mec}$  remain constant. The “on-load” or the “short-circuit” losses  $p_{sc}$  are stator current related, and the excitation losses  $p_{exch}$  vary with their currents squared:

$$p_{sc} = p_{scn} \left( \frac{I_1}{I_n} \right)^2 ; \quad K_{load} = \frac{I_1}{I_n} \quad (7.254)$$

$$p_{exch} = p_{exchn} \left( \frac{I_f}{I_{fn}} \right)^2 + p_{brush} \cdot \left( \frac{I_f}{I_{fn}} \right) ; \quad P_2 = S_n \cdot \frac{I_1}{I_{1n}} \cdot \cos \varphi$$

So, the efficiency at part load is

$$\eta_{SG} = \frac{K_{load} \cdot S_n \cdot \cos \varphi}{K_{load} \cdot S_n \cdot \cos \varphi + p_{Feo} + p_{mec} + p_{scn} \cdot K_{load}^2 + p_{exch} \cdot \left( \frac{I_f}{I_{fn}} \right)^2 + p_{brush} \cdot \left( \frac{I_f}{I_{fn}} \right) + p_{stray}} \quad (7.255)$$

Stray load losses are added.

Knowing all steady-state machine parameters, for each value of output power and  $\cos \varphi$ , stator and field currents may be calculated after the power angle  $\delta_v$  is determined along the way (Chapter 4). With losses calculated as in previous paragraphs, the total losses and efficiency at different load levels may be predetermined to assess the design goodness from this crucial point of view.

Note that in direct-cooling SGs, the winding losses tend to dominate the losses in the machine, while for indirect cooling, the nonwinding losses tend to be predominant.

It follows that for indirect cooling, the efficiency tends to be maximum above rated load and below rated load for direct cooling.

## 7.20 Exciter Design Issues

The excitation system supplies the SG field current to control either the terminal voltage or the reactive power to set point.

As already detailed in [Chapter 6](#), the excitation system is provided with protective limiters.

For operation tied to weak power systems, power system stabilizers (PSSs) are used to dampen the oscillations in the range from 0.5 Hz to  $2 \div 5$  Hz.

The specification of excitation systems is guided by IEEE standards 421 [16]. There are two key factors that define an excitation system: the transient gain and the ceiling forcing ratio (maximum/rated voltage at SG excitation winding terminal). The transient gain has a direct impact on small signal and dynamic stability. Too small a transient gain may fail to give the desired performance, while too high a transient gain produces faster response but may result in dynamic instability during faults. In this latter case, the PSS may have to be considered. The transient gain refers to frequency of generator oscillations in the range of a few tenths of hertz to a few hertz.

Traditionally, excitation system controllers consisted of a high steady-state gain to secure low steady-state control error and a lag-lead compensator to provide transient gain reduction.

In today's digital control systems PI, PID or more advanced controllers such as Fuzzy Logic,  $H_\infty$ , sliding-mode, etc. are used.

Steady-state gains of 200 P.U. provide for  $\pm 0.5\%$  steady-state regulation.

Transient gains vary, in general, from 20 to 100 with lower values for weak power systems, to avoid local mode power system oscillations (in the 1 to 2 Hz range).

The excitation system ceiling voltage also has an impact on transient stability.

For static excitation systems, 160 to 200% of rated field voltage is set for design to preserve stability after the clearing of a three-phase fault on the higher voltage side of the SG step-up transformer.

As lowering the SCR seems to be the trend in SG design today, to increase SG efficiency, increasing the excitation voltage ceiling becomes the pertinent option to preserve transient stability desired performance.

The three-phase fault clearing critical time (CCT) for constant ceiling forcing decreases when the SCR decreases, both for a lagging and for a leading power factor. For leading power factor, however, the CCT is even smaller.

The required field voltage ceiling ratio has to be specified after critical clearing time limits are checked for lagging and leading power factor for the actual power system area where the SG will work. Such a local system is characterized by the percent reactance looking out from the SG: 10% is a strong system and 40% is already a weak system.

Practical limits of ceiling (maximum) voltage are about  $500 V_{dc}$  in general. But this maximum voltage may correspond to a 3 P.U. ceiling voltage ratio.

AC brushless exciters might allow for a smaller ceiling voltage ratio, as they are less affected by power system faults.

In respect to exciter design, we will treat here only the AC brushless exciter, as it comprises two synchronous generators of smaller power rating.

An alternative brushless exciter, operational from zero speed, is made from a wound rotor induction machine with rotor winding output that is diode rectified and supplies the SG excitation, while its stator winding is supplied at variable (controlled) voltage through a thyristor voltage amplitude controller ([Figure 7.35a](#)).

The frequency  $f_2$  of the voltage induced in the AC exciter rotor is  $f_2 = f_1 + n \cdot p_{lex}$  and increases with speed, above  $f_1 = 50(60)$  Hz.

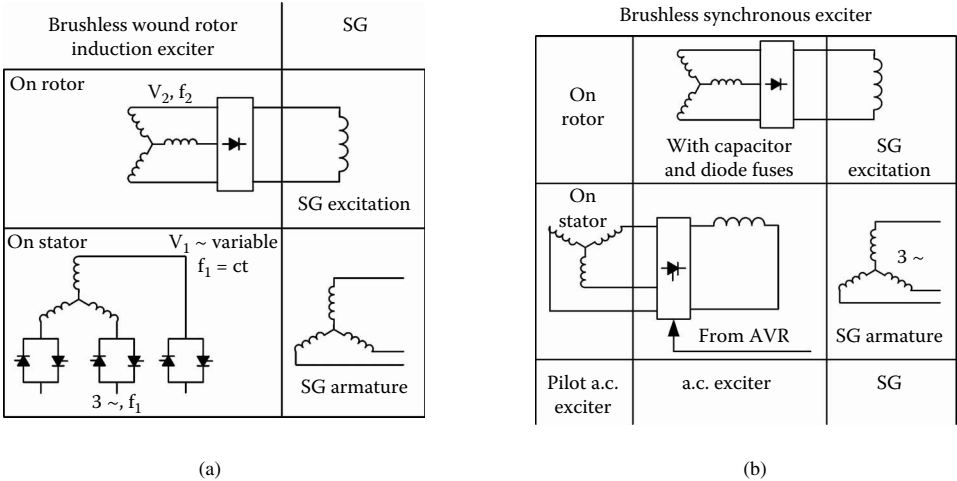


FIGURE 7.35 Brushless alternating current exciter: (a) induction exciter and (b) synchronous exciter.

As the induced rotor voltage amplitude  $V_2$  also tends to increase with  $f_2$ , when speed increases, the voltage  $V_1$  in the stator has to be reduced through adequate control.

At zero speed, all the power transmitted to the SG excitation comes from the exciter’s stator; it is electrical, in other words.

As the speed increases, more of the excitation power comes from the shaft, especially with increasing the exciter’s pole pair number  $p_{1exc} > 3$ .

The induction exciter voltage response is expected to be rather fast. However, the exciter stator winding has to deliver the whole power of SG excitation, should it be needed at zero speed, as in the case of a pump-storage plant. The induction exciter is also used in industry.

A typical AC brushless exciter contains a PM synchronous pilot exciter with output that is electronically controlled after rectification to supply the DC excitation of the inside–out synchronous exciter with a diode rectified output (with the rectification on the rotor) that is connected directly to the excitation circuit of the SG (Figure 7.35b).

This way, no brushes or an additional power source is involved. However, if faster response is needed, the excitation circuit of the pilot exciter may be supplied directly from a separate power source through power electronics, with a control variable that is the output of the SG excitation control.

The pilot PM alternator is designed for frequencies in the range of 180 to 450 Hz to cut the rectified output voltage pulsations. Also, the AC exciter is designed with a rather large number of poles  $2p_1 = 6 - 8$ , again to secure a rated armature frequency higher than 100 Hz; thus, low pulsations in the field-winding current and small volume of the AC exciter magnetic circuit (armature yoke, especially) are found.

The relationship between AC exciter phase voltage per phase  $V_{1s}$  (RMS value) and the rectified field-winding voltage  $V_f$  is as follows:

$$V_{1s} \approx V_f / 2.34$$

(7.256)

$$I_{1s} \approx 0.780 \cdot I_f$$

(7.257)

The voltage regulation of the diode rectifier in SG excitation circuit is neglected. If better precision is needed, rectifier voltage regulation has to be considered, as in Chapter 6 (Figure 6.33).

Equation 7.256 and Equation 7.257 imply an ideal (lossless) rectifier.

The AC exciter should be available to produce the ceiling voltage  $V_{fmax}$  at the SG excitation terminals, also, during steady state, to allow for the  $I_{fn}$  (the rated load excitation current that corresponds to rated SG output power and power factor).

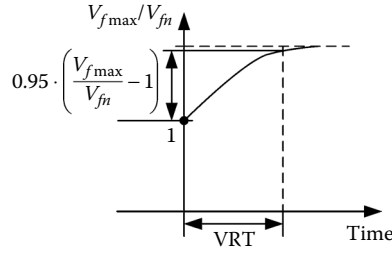


FIGURE 7.36 Voltage response time (VRT).

Besides rated and maximum field voltage and current ratings  $V_{fn}$ ,  $I_{fn}$  for steady state and  $V_{f\max}$ ,  $I_{f\max}$  for transients, the AC exciter has to provide a certain response time in providing the ceiling voltage  $V_{f\max}$  from the rated value  $V_{fn}$ .

Equally important, the excitation system has to have high efficiency at moderate weight and costs and be reliable. Here, a general design method for a brushless AC exciter is detailed. Optimization design issues will be treated later in this chapter for the SG in general.

### 7.20.1 Excitation Rating

As already alluded to, the AC exciter rating is given by  $V_{fn}$ ,  $I_n$  for steady state, and  $V_{f\max}$ ,  $I_{f\max}$  for transients.

The exciter response time is also specified. The SG may be required to operate at rated megavoltampere for 105% terminal voltage. As in such a case the magnetic saturation level in the SG increases notably, the continuous power rating of the AC exciter will increase more than 5%. Sometimes, the exciters have redundant circuitry.

The voltage response time (VRT) is the time in seconds for the excitation voltage to travel 95% of the difference between  $V_{f\max}$  and  $V_{fn}$  (Figure 7.36).

Today's AC exciter systems are required to have  $\text{VRT} < 100$  msec under full field current load.

### 7.20.2 Sizing the Exciter

Sizing the exciter is similar to that process of the main SG, as already developed in this chapter. There are some peculiarities, though [17].

The reduction in voltage response time leads to severe limitation in the AC exciter subtransient reactance  $x'_d$  below 0.3. It may also be needed to provide for a rather strong cage in its stator to reduce the commutation reactance  $x_c = (x''_d + x''_q) / 2$  in the diode rectifier for lower voltage regulation and allowance for higher rated frequency (low yoke height).

As a good part of  $x'_d$  is the armature leakage inductance, reducing the latter is paramount. Wide and not so deep armature slots and short end connections (chorded coils) are practical ways to achieve this goal. Also, the leakage inductance of the AC exciter circuit has to be reduced, star connection of AC exciter armature phases is preferred in order to eliminate the third harmonic current and thus avoid limitations on coil span. It also requires a smaller number of slots (with two turns/slot) for the same output DC voltage  $V_{fn}$ .

A smaller number of slots means lower manufacturing costs.

Two or more current paths in parallel may be necessary to produce the required  $V_{f\max}$  ( $V_{fn}$ ).

### 7.20.3 Note on Thermal and Mechanical Design

So far, the various no-load and load losses were calculated. To remove the heat produced by losses, it is necessary to adopt and design appropriate cooling systems.

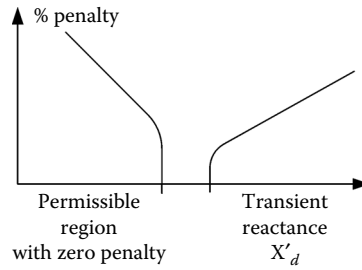


FIGURE 7.37 Penalty function for  $X'_d$ .

As already mentioned, SG cooling may be indirect (with air or hydrogen) and direct (with water or hydrogen). Evaporative cooling is gaining in popularity, as well. There are many specific solutions as the power and speed (in rpm) goes up.

Analytical models are generally used for thermal design, but FEM is increasingly used for the scope. The same rationale is valid for the mechanical design.

As the thermal and mechanical limitations observation influences the machine reliability and safety, their summarily treatment is not to be accepted. A detailed such analysis, on the other hand, needs rich space. This is why we stop here, drawing attention to specialized design books [5, 6], while noticing that all manufacturers have their proprietary thermal design methods for SGs.

Forces, noise, and vibration are other important issues in SG design. See, for starters [6, 18].

## 7.21 Optimization Design Issues

So far, we presented analytical expressions to do the dimensioning of an SG that fulfills given specifications. Also, the computation of resistances, inductances, and losses was investigated. Finally, the cost of active materials may be computed.

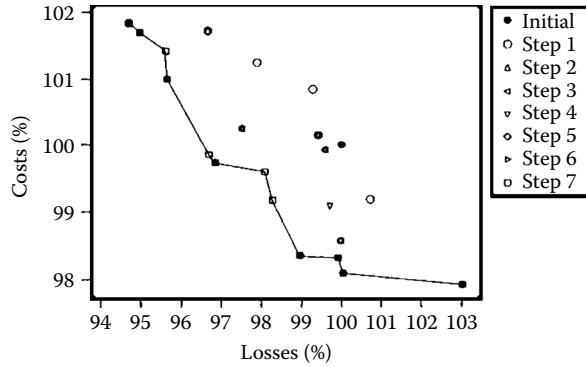
This process may be described as the general design, and computer programs may be produced to mechanize it.

The general design produces a realistic (feasible) SG. It is a good starting point for optimization design. Optimization design requires the following:

- An initial practical design
- A machine model to calculate the parameters and performance (and objective functions) for given geometry and various design parameters
- A set of design variables, such as the following:
  - Stator bore diameter is  $D_{is}$
  - Slot width is  $W_{ss}$
  - Slot/tooth width ratio is  $W_{ss}/W_t$
  - $A$  (A/m) is the stator current loading
  - $W/m^2$  is the stator surface loading
  - Flux density in stator teeth:  $B_{ts}$
  - Flux density in stator yoke:  $B_{ys}$
  - Flux density in the rotor pole body:  $B_p$
  - Thickness of field coil:  $W_c$
  - $W/m^2$  rotor surface loading

For each variable, minimum and maximum values are set:

- A single or multiple cost function, such that generator costs plus the loss of capitalized costs is a way to accommodate various constraints (such as  $X'_d < X'_{dmax}$ , various temperature limitations,  $SCR < SCR_{max}$ , etc.) through penalty terms added to the cost function (Figure 7.37).



**FIGURE 7.38** Pareto front (squares and line) and the initial solution (solid circle).

- A mathematical method (or more) to search for the optimal design [19] that produces the lowest cost function, observing the constraints, and being able to deal with some design variables that are integer numbers (number of slots  $N_s$ , number of turns in the coil, etc.).
- There is a plethora of optimization methods of linear and nonlinear programming, indirect and direct, with constraints, deterministic, stochastic, and evolutionary plans.

Such a complete optimization design software, with long use, based on the Monte Carlo optimization method, is described in Reference [20].

More recently, evolution strategies and genetic algorithms (G.A.) were applied to optimize the subdesign of SGs. By subdesign, we mean here a partial problem in design, such as the modification of rotor pole or of stator slot geometry to minimize losses and production costs [21].

When the cost (objective) function may not be stated in terms of a scalar-type function, Pareto optimal sets are used. Pareto sets contain a number of solutions (designs) that are equivalent to each other in terms of trade-offs, that are so common in SG design.

Finding Pareto optimal sets of solutions to the problem, with stator slot and rotor pole dimensions as design variables, and losses and production costs to be simultaneously minimized, constitutes such an optimization subdesign problem in SGs. For such a problem, the design variables could be as follows:

- Geometry and number of copper strands
- Dimensions of rotor pole
- Dimensions of field coil
- Pitch and diameter of damper cage bars
- Airgap

Typical constraints to be enforced through penalty functions are as follows:

- $X'_d < X'_{d\max}$ ,  $X''_d > X''_{d\min}$
- Temperature rise
- Clearance between adjacent field coils
- Mechanical stresses in rotor poles and their dovetails
- Admissible flux density and current density limits

Evolutionary method algorithms of optimization become necessary in nonsmooth and model-based objective (cost) functions. This is the case for SGs.

Figure 7.38 [21] shows the Pareto front (squares and line) with the initial solution (solid circle), while the other points are dominated solutions.

Two simultaneous goals — minimum cost and minimum losses (5000 \$/kW of losses) — were followed in this case.

The G.A. were proven to produce better results [20] than evolutionary strategies.



## 7.22 Generator/Motor Issues

Pump storage is used in some hydropower plants to replenish the water in the forebay each day and then deliver electric power during the peak demand hours. During the pumping mode, the SG acts as a motor. To start such a large power motor, even on no load, a pony motor was traditionally used. It is also practical to use the back-to-back generator and motor starting when a unit works as a generator and, during its acceleration to speed, it supplies a motor unit and the two advance synchronously to the rated synchronous speed.

A single generator may supply one after the other all pump-storage units. An induction-motor-mode starting with self-synchronization is also feasible, at reduced voltage for large power units.

As the friction torque may be reduced, by refined high-pressure oil bearings, to less than 1% of rated torque, the power required to start the motor, with the waterless turbine — pump, was notably reduced.

It is thus feasible to use a lower relative power rating static power converter to start the motor and disconnect the converter after the motor has been self-synchronized.

An SG designed to be used also as a motor for water pump storage undergoes about one start a day as a motor and one synchronization a day as a generator.

Self-synchronization for motoring and automatic synchronization for generation imply notable transients in currents and torque, and the machine has to be designed to withstand such demanding conditions.

Typically, the direction of motion is changed for motoring as required by the turbine pump. High wicket gate water leakage may cause the “waterless” unit to start rotating still in the generator direction. To avoid the transients that the static power converter would incur in initiating the starting as a motor, the machine has to be stalled before beginning to accelerate it on no load, in the motoring direction.

Asynchronous self-starting is used for low and medium power units, albeit using a step-down autotransformer or a variable reactor to reduce starting currents. For units above 20 MW, pony motor, generator motor back to back, or static power converter starting should be used.

The development of multiple-level voltage source converters with IGBTs (up to 4.5 MW/unit) [22] and with GTOs or MCTs (up to 10 MW/unit and above) [23] might change the picture.

For the low power range (up to 3 MW), it is possible to use a bidirectional voltage source IGBT converter, designed at a generator motor rating, that will not only allow startup for the motoring, but will also provide for generating and motoring at variable speed (to exploit optimally the turbine — pump). It is known that the optimal speed ratio between pumping and turbinning is about 1.24. For high power, the multilevel voltage source GTO and MCT converters may replace the controlled — rectifier — current source converters that are used today to accelerate pump-storage units (above 20 MW or so) and then self-synchronize them to the power grid.

The multilevel PWM back-to-back IGBT, GTO, MCT converters provide quality starting while inflicting lower harmonics both on the power line and on the motor side (Figure 7.39a and Figure 7.39b). They are smaller in size and provide controllable (unity) power factor on the line side.

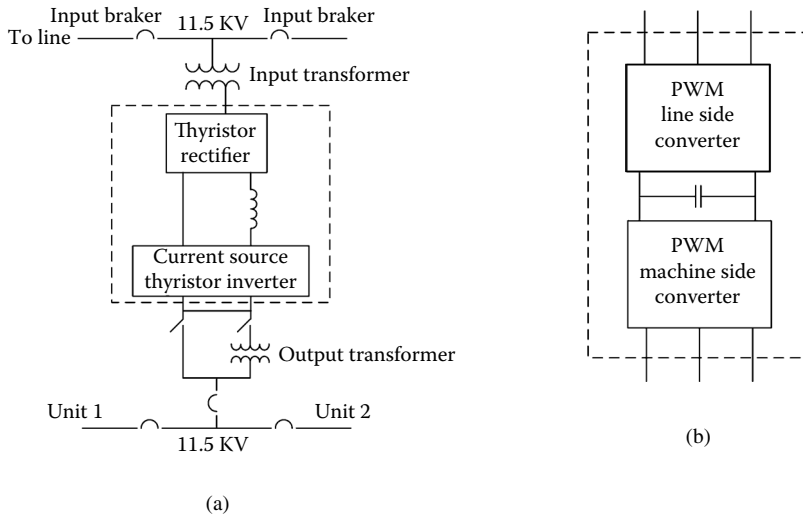
As there is a step-up transformer between the static power converter and the machine, the starting cannot be initiated from zero frequency but instead, from 1 to 2 Hz. This will cause transients that explain the necessary oversizing of the static power converter for the scope.

It goes without saying that to start the motor as a synchronous motor, a static exciter is required, because full excitation is needed from 1 to 2 Hz speed. The static converters might be controlled for asynchronous starting (up to a certain speed), and thus, their kilovoltampere rating has to be increased.

Note that the “ideal” solution for pump storage is the doubly fed induction generator motor that will be studied in *Variable Speed Generators*.

## 7.23 Summary

- By SG design, we mean here dimensioning to match given specifications of performance with constraints.



**FIGURE 7.39** Static converter starting in pump-storage plants: (a) with rectifier-current (one-phase) source inverter and (b) with voltage source converter.

- Specifying SGs to be connected to a power system is an object of standardization. The proposed consolidated C 50.12, C 50.13 ANSI standards are typical for the scope.
- Besides manufacturers' standards, grid codes have been issued to tailor SG performance to power system typical and extreme operation modes. There may be conflicts between those two kinds of specifications, and mitigation efforts are required to harmonize them.
- The first specification item is the short-circuit ratio (SCR) — the ratio between the short-circuit and rated current or the inverse of  $d$  axis synchronous reactance in P.U. ( $1/x_{dsat}$ ). A larger SCR requires a larger airgap and, finally, a larger excitation mmf and losses, while it increases the peak power and thus the static stability. As the transmission line reactive  $x_c$  at generator terminal increases, the beneficial effect of a larger SCR on static stability diminishes. A large  $x_c$  characterizes a weak power system. Today's trend is to reduce SCR to  $0.4 \div 0.6$ , while preserving the static stability by the ever-faster response of contemporary exciter systems.
- The second specification issue is the transient reactance  $x'_d$  that influences the transient stability. The smaller  $x'_d$ , the better, but there are limits to it due to machine geometry required to produce the design power.
- Rated power factor is essential in SG design: the lower the rated power factor, the larger the excitation power. In general,  $\cos\phi_n$ : 0.9 to 0.95 (overexcited).
- The maximum absorbed (leading) reactive power limit is determined by the SCR and corresponds to maximum power angle and to the stator end-core temperature limit.
- Fast control of excitation current is required to preserve SG stability and control the terminal voltage. Today's ceiling voltage ratio for excitation systems is 1.6 to 3.0. The limit is, in fact, determined by the heavy saturation of an SG magnetic circuit. A response time of 50 msec "producing" the maximum ceiling voltage is required to qualify the response of the excitation system as fast. Even with brushless rotary machine configurations, such a response is feasible. Static exciters are much faster.
- Voltage and frequency variations are limited to contain the saturation level in the SG. A  $\pm 2$  to 3% in both should be provided at rated power. Higher values are to be accepted for a limited amount of time to avoid SG overheating. These requirements are standardized or agreed upon between manufacturers and users.
- Negative phase-sequence voltages and currents are limited by standards to limit rotor damper cage and excitation additional losses (and overtemperatures). Negative sequence voltage is limited to

about 1% ( $V_2/V_1\%$ ), while the current negative sequence  $i_2/i_1$  may reach 4 to 5% and depends on the SG negative sequence reactance  $x_2$  plus the step-up transformer short-circuit reactance  $x_T$ .

- Total harmonic distortion (THD) limits are prescribed by grid codes to 1.5 to 2%, with proposals to raise them to 3 to 3.5% in the future.
- The temperature basis for rating power is paramount in SG design. It is between observable and hot-spot temperature. The observable temperature limit is set such that the hot-spot temperature is below 130°C for class B and 155°C for class C insulation systems. The rated coolant temperature also has to be specified. As the cold coolant temperature varies for ambient-following SGs, it seems reasonable to fix the observable temperature limit for a single cold coolant temperature and calculate the SG megavoltampere capability for different ambient temperatures. The hot-spot temperature will also vary. As the ambient temperature goes up, the SG megavoltampere capability goes down.
- Start–stop cycles have to be specified to prevent cyclic fatigue degradation. For base load SGs, 3000 starts are typical, while 10,000 starts are characteristic to peaking load or other frequently cycled units.
- Starting and operating as a motor (in pump-storage units), faulty synchronizations, forces, armature voltage, and runaway speed ratio, have to be specified in a pertinent design.
- The design process consists of sizing the SG in relation to the above-mentioned specifications. The design refers to electromagnetic, thermal, and mechanical aspects. Only electromagnetic design was followed in detail in this chapter. The electromagnetic design essentially comprises the following:
  - The output coefficient stator geometry
  - The selection of number of stator slots
  - The design of the stator core
  - The sizing of salient pole rotors
  - The sizing of cylindrical pole rotors
  - Open-circuit saturation curve computation
  - Stator leakage reactance and resistance computation
  - Field mmf (current) at full load
  - Computation of synchronous reactances  $x_d$ ,  $x_q$
  - Sizing of damper cage
  - Computation of time constants and transient, subtransient reactances
  - Exciter design
  - Generator/motor design issues
  - Optimization design issues
- The output coefficient  $C$  (min kVA/m<sup>3</sup>) is defined as the SG in kilovoltampere per cubic meter. It is a result of accumulated experience (art) and varies with the power/pole, number of poles pairs, and type of cooling system. The rotor diameter  $D$  is computed basically based on runaway speed  $U_{\max}$ , with given SG synchronous speed and number of pole pairs  $p_1$ , for given frequency. The rotor inertia  $H$  (in seconds) has to be above a limit value so as to limit the maximum speed of the SG upon loss of electric load, until the speed governor closes the fuel (water) input (unless special fast braking is provided, as is the case of turbogenerators).
- SGs may have integer or fractionary two-layer windings. We refer to  $q$  slots/pole/phase as integer or fractionary. Turbogenerators ( $2p_1 = 2, 4$ ) make use of integer  $q$  windings in general ( $q > 5$  to 6), while hydrogenerators today have fractionary windings. The choice of  $q$  determines the number of slots  $N_s$  of the stator as  $N_s = 6 \cdot p_1 q$ . The selection of  $q$  also depends on the number of lamination segments  $N_c$  that constitute the stator core and on the number of detachable sections  $N_K$  in which the core may be divided and wound separately due to the large diameters considered:
  - $N_K = 2$  for  $D < 4$  m
  - $N_K = 4$  for  $D = 4 \div 8$  m
  - $N_K = 6(8)$  for  $D > 8$  m

It is imperative that all  $N_c$  segments spread over an integer number of poles  $N_{ss}$ . For even  $N_{ss}$ ,  $N_s$  is even and integer  $q$  is feasible. For fractionary  $q$ ,  $N_{ss}$  may be an odd number and contain three as a factor.

- For fractionary  $q = b + c/d$ , symmetrical windings are obtained, in general, for  $d/3 \neq \text{integer}$ . Windings are made with wave coils and one turn per coil and with lap coils when two or more turns per coil are used.
  - For wave windings,

$$\frac{3c \pm 1}{d} = \text{integer}$$

- The noise level is reduced if

$$3 \left( b + \frac{c}{d} \right) \pm \frac{1}{d} \neq \text{integer}$$

with  $c/d = 1/2$  ( $b > 3$ ), the subharmonics are eliminated.

- Apparently, once the value of  $q$  is settled, the number of turns per current path is obtained from the stator geometry and the total rated emf  $e_t \approx 1.03$  to 1.10 P.U. For one turn (bar) per coil, the number of slots is directly related to the number of turns/path  $W_a$  and number of current paths  $a$ :

$$N_s = a \cdot W_a \cdot 3$$

It is now easy to see how crucial the choice of  $N_s$  is as  $W_a$  (integer) comes directly from the emf  $e_t$  value.

- To size the stator core, the airgap  $g$  has to be computed first, as the rotor diameter  $D_r$  is already known. Sizing  $g$  is directly related to stator-rated linear current loading  $A$ , pole pitch  $\tau$ , airgap flux density fundamental  $B_{g1}$ , and the desired  $x_d$  (SCR).
- The rated voltage increases with the SG power/unit and is larger for direct cooling. It is, otherwise, decided by the manufacturer. In general,  $V_{\text{line}} < 28$  kV, but cable winding 40 to 100 kV SG are in the advance development stage. They lead to the elimination of the step-up transformer that accompanies practically all SGs tied to power systems.
- The salient pole rotors are designed for variable airgap under the pole, to render the airgap flux density more sinusoidal. The maximum to minimum airgap  $g_{\text{max}}/g$  ratio is up to 2.5. In practice, the radius of the pole shoe shape  $R_p$  is notably smaller than rotor radius  $D/2$ .
- Damper cage design is based on the main assertion that the cage bars total cross area represents 15 to 30% of stator slot area. The damper cage slot pitch  $\tau_d$  is chosen to reduce losses in the damper cage due to stator mmf space harmonics. For fractionary stator windings ( $q = (bd + c)/d$ ) when  $bd + c > q$ , it is feasible to chose  $\tau_s = \tau_d$  ( $\tau_s$ , stator slot pitch). For  $bd + c < q$  or  $q = b + 1/2$ ,  $\tau_s \neq \tau_d$ . The damper bars of copper or brass are welded to end-rings with a cross-section that is about half the bars cross-section per rotor pole.
- The cylindrical rotor, made of single or multiple (axially) solid iron rings, is slotted along about two thirds of its periphery to host the field coils. Lower airgap flux harmonics are obtained by placing the field coils in slots, while this is also the only mechanically feasible solution to withstand the centrifugal force stresses. To design the field winding, the rated field mmf is required. As at this stage in design this is not known, a value of 2 to 2.5 times the rated stator mmf per pole is adopted.
- The open-circuit saturation curve may be obtained through an analytical model accounting for magnetic saturation or through FEM.
- The rated field mmf is obtained traditionally through the Potier diagram (Figure 7.24) or through the partial magnetization curves (Figure 7.25). The second method allows for cross-coupling

saturation, while the first one does not and is adequate for cylindrical rotor SGs. Based on the partial magnetization curves method, the synchronous reactances  $x_d$  may be determined as functions of  $i_d$  and  $i_q$  as families of curves. Alternatively,  $\Psi_d(i_d, i_q)$  and  $\Psi_q(i_d, i_q)$  may be obtained. Such information is essential in calculating with good precision the rated power angle and steady-state performance of SGs.

- The computation of rotor resistances and various leakage inductances is straightforward if approximate formulae are accepted. They proved adequate in industry, though today, they may be computed by FEM also.
- The solid rotor behaves as an equivalent damper cage with variable parameters. A reliable analytical model to calculate these parameters is presented. This model was recently validated by FEM.
- The various time constants  $T'_{do}, T_{do}, T'_d, T''_d, T_a, T_D$  and the transient and subtransient reactances (in P.U.) are straightforward once the synchronous reactance  $x_d, x_q$  and all leakage reactances and resistances are defined by analytical expressions. Note that  $x'_d < x'_{d\max}$ . The negative sequence reactance  $x_2$  is also calculated easily as follows:

$$x_2 = (x''_d + x''_q)/2 \quad \text{or} \quad x_2 = \sqrt{x''_d \cdot x''_q}$$

- Emf time harmonics are limited by standards. Higher  $q$ , coil chording, and fractionary  $q$  are used to reduce emf time harmonics. The variable airgap obtained by shaping the rotor salient poles reduces the fifth and seventh harmonics, while it introduces the third harmonic at the 10 to 14% level.
- The interaction of field mmf fundamental with stator slot openings may produce the so-called slot-ripple emf effect, that is, additional damper cage losses at no load. With  $N_s/p_1$  as an odd number, ideally zero damper cage additional losses at no load occur.
- There are various losses in an SG, but they may be grouped into no-load and short-circuit electromagnetic losses and mechanical losses.
- Analytical formulae are given for most components of losses, and, finally, the efficiency expression is obtained. The designer may thus calculate the total efficiency for various load conditions. Efficiency and losses are paramount performance indexes in an SG.
- Exciter design specifications and issues are treated in some detail for the brushless exciter: note the voltage response time for the excitation voltage to reach 95% of the ceiling (maximum) value (starting from the rated one).
- Once the general design is completed, a sound initial solution exists, and a reliable model of the machine is in place. Optimization design may start, once a set of variables is defined, a cost function is derived, and a mathematical optimization algorithm is chosen. Results with evolutionary and genetic algorithm methods are presented from the literature.
- For generator/motor units, a detailed discussion of motor starting and synchronization methods is given. The unit has to withstand the daily start-stop encountered in generator/motor cycles for pump-storage operation modes.
- Numerical examples are given throughout this chapter to illustrate the various design issues.

## References

1. B.E.B. Gott, W.R. Mc Cown, and J.R. Michalec, Progress in Revision of IEEE/ANSI C 50 Series of Standards for Steam and Combustion Turbine Generators and Harmonization with the IEC 60034 Series, Record of IEEE-IEMDC-2001 Conference, MIT.
2. R.J. Nelson, Conflicting Requirements for Turbogenerators from Grid Codes and Relevant Generator Standard, Record of IEEE-IEMDC-2001, Meeting, MIT, pp. 63–68.
3. C.E. Stephan, and Z. Baba, Specifying a Turbogenerator's Electrical Parameters by Standards and Grid Codes, Record of IEEE-IEMDC-2001, Meeting, MIT, pp. 57–62.

4. J.M. Forgarty, Connection Between Generator Specifications and Fundamental Design Principles, Record of IEEE-IEMDC-2001, Meeting, MIT, pp. 51–56.
5. K. Vogt, *Electrical Machines*, VEB Verlag Technik, Berlin, 4th ed., 1988, pp. 416 (in German).
6. J.H. Walker, *Large Synchronous Machines*, Clarendon Press, Oxford, 1981.
7. V.V. Dombrowski, A.G. Ereemeev, N.P. Ivanov, P.M. Ipatov, M.I. Kaplan, and G.B. Pinski, *Design of Hydrogenerators*, vol. I and II, Energy Publishers, Moskow, 1965 (in Russian).
8. V. Ostovic, *Dynamics of Saturated Electric Machines*, Springer-Verlag, Heidelberg, 1989.
9. R. Richter, *Electric Machines*, vol. 2, Verlag Birkhäuser, Basel, Stuttgart, 1963 (in German).
10. I. Boldea, and S.A. Nasar, *The Induction Machine Handbook*, chap. 6, CRC Press, Boca Raton, FL, 2001.
11. A.B. Danilevici, V.V. Dombrovski, and E.I.A. Kazovski, A.C. *Machinery Parameters*, Science Publishers, Moskow, 1965 (in Russian).
12. I.S. Gheorghiu, and A. Fransua, *Treatise of Electric Machines*, vol. 4, *Synchronous Machine*, Tech. Publ., Bucharest, 1972 (in Romanian).
13. E. Levi, *Polyphase Motors: A Direct Approach to Their Design*, John Wiley & Sons, New York, 1985.
14. K. Oberretl, Eddy current losses in solid pole shoes of synchronous machines at no-load and on load, *IEEE Trans.*, PAS- 91, 1972, pp. 152–160.
15. O. Drubel, and R.L. Stall, Comparison between analytical and numerical methods for calculating tooth ripple losses in salient pole synchronous machines, *IEEE Trans.*, EC-16, 1, 2001, pp. 61–67.
16. A. Murdoch, and M.J. D'Antonio, Generator Excitation Systems — Performance Specification to Meet Interconnection Requirements, Record of IEEE-IEMDC-2001, MIT, June 2001.
17. M. Tartibi, and A. Domijan, Optimizing A.C. Exciter design, *IEEE Trans.*, EC-11, 1, 1996, pp. 16–24.
18. A.M. Knight, N. Karmaker, and K. Weeber, Use of a permeance model to predict force harmonic components and damper winding effect in salient-pole synchronous machines, *IEEE Trans.*, EC-17, 4, 2002, pp. 478–484.
19. S.S. Rao, *Optimization Methods in Engineering*, John Wiley & Sons, New York, 1996.
20. O.W. Andersen, Optimized Design of Salient Pole Synchronous Generators, Record of ICEM-200, Espoo, Finland, August 28–30, 2000, pp. 987–989.
21. E. Schlemmer, W. Harb, G. Lichtenecker, F. Muller, and R. Kleinhaentz, Optimization of Large Salient Pole Generators Using Evolution Strategies and Genetic Algorithms, Record of ICEM-2000, Espoo, Finland, pp. 1030–1034.
22. J. Bäcker, J. Janning, and H. Jebenstreit, High dynamic control of a three-level voltage–source–converter drive for a main strip mill, *IEEE Trans.*, EC-12, 1, 1997, pp. 66–72.
23. T.A. Meynard, H. Foch, P. Thomas, J. Courault, R. Iakob, and M. Mahrstaedt, Multilevel converters: basic concepts and industry applications, *IEEE Trans.*, IE-49, 5, 2002, pp. 955–964.